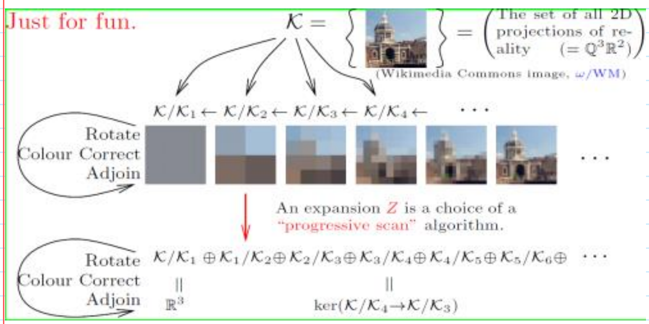
# 17-1350-AKT Tue Feb 14, Gentle Hours 11-12: Finite Type Invariants

February 7, 2017 8:32 AM

0. I'm not totally sure what comes next....

1. Filtered & graded vector spaces, the functors gr & fil, expansions.




Include progressive scanning material, though use the example at 170214-ProgressiveScanning.nb:

Pensieve header: Progressive scanning of a photo.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\17-1350-AKT"];
img = ImageData@Import["CP8.png"];
SlideView@Table[bs = 2^(8 - k);
  Graphics[Raster[Reverse[Total[Partition[img, {bs, bs}], {3, 4}] / bs^2.]],
   ImageSize → 256, ImagePadding → None, PlotRangePadding → None],
  {k, 0, 8}]
```

Aside: expansions always exist.

Proof $\frac{F_n}{F_{n+1}} \hookrightarrow \frac{F_0}{F_{n+1}}$, $\frac{F_n}{F_{n+1}} \xleftarrow{p_n} \frac{F_0}{F_{n+1}}$, so find a projection $p_n$ going the other way. With $\pi_n : V \to F_{n+1}V$ the quotient map, $\prod_n p_n(\pi_n(v))$ is an expansion.

2. homomorphic expansions.

3. Example $R = C^\infty(\mathbb{R}^n)$ $I = \{f : f(0) = 0\}$, ....

4. Example. Expansions for groups.

5. Filtrations on tangles. $\mathcal{T}_n = \{\underbrace{\times \cdots \times}_{n}\}$

6. The dual perspective: F.T. invariants: $V$ s.t. $V/_{\mathcal{T}_{n+1}} = 0$.

   1. like polynomials.
   2. Plenty examples. $\nearrow\!\!\!\!\nwarrow \mapsto q\ )( - q^2 \asymp$ $\nwarrow\!\!\!\!\nearrow \mapsto -q^{-2} \asymp + q^{-1}\ )($ $\bigcirc \mapsto q + q^{-1}$
      so $q^{-1} \nearrow\!\!\!\!\nwarrow - q \nwarrow\!\!\!\!\nearrow = (-q + q^{-1}) \asymp$ .....
   3. $V_n = (\mathcal{T}_0/\mathcal{T}_{n+1})^*$ so $V_n/V_{n-1} = (\mathcal{T}_0/\mathcal{T}_{n+1})^*/(\mathcal{T}_0/\mathcal{T}_n)^* = (\mathcal{T}_n/\mathcal{T}_{n+1})^*$

done line

7. $\mathcal{A}^u(\mathcal{T})$: chord diagrams, 4T, FI

8. Decirfication: A "guess gr" is a graded $A = \bigoplus A_n$ w/ graded $\pi : A \to \operatorname{gr} V$

   An $A$-expansion is $Z_A : V \longrightarrow A$ s.t. $\pi // \operatorname{gr} Z_A = Id : A \to A$

   meaning, if $a \in A_n$, $\pi a = [f]$ w/ $f \in F_n V$, $Z_A(f) = (0 \ldots 0, a, *, *)$.

   Claim If $Z_A : V \to A$ is an $A$-expansion, then guess is confirmed & $Z = Z_A // \pi$ is an expansion.

   Pf
$$\begin{array}{c} \xrightarrow{Z_A} A \xleftarrow{\operatorname{gr} Z_A} \\ \downarrow \pi \end{array}$$

Pf

$$\begin{tikzcd} & A \arrow[d, "\pi"] \\ V \arrow[ur, "Z_A"] \arrow[r, "\pi\circ Z_A"'] & \operatorname{gr} V \arrow[u, dashed, bend right, "\operatorname{gr} Z_A"'] \end{tikzcd}$$