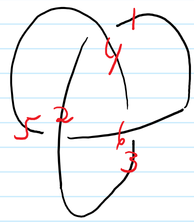
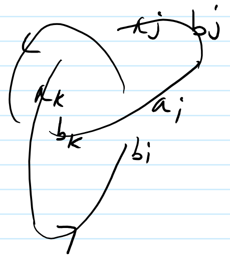
17-1350-AKT Fri Jan 20, Brute Hour 2: Implementing g0

January 19, 2017 2:20 PM

No HW!



The scheme: start w/ $R \in A \otimes A = \hat{U(g)} \otimes \hat{U(g)}$ s.t. $R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12}$, get knot invariants in $A = U(g)$ by placing a copy of R at each crossing & multiplying along:



$R = \sum a_i \otimes b_i$

$\longrightarrow \sum_{i,j,k} b_i a_j b_k a_i b_j a_k \in U(g)$

PBW Theorem: If $x_1 \dots x_k$ is a basis of g, then $\{x_1^{a_1} x_2^{a_2} \dots x_k^{a_k} : a_i \in \mathbb{Z}_{\geq 0}\}$ is a basis of $U(g)$. So as vector spaces $U(g) \cong S(g) \cong \mathbb{Q}[x_1 \dots x_k]$, though $U(g)$ has a funny product.

Today: $g_0 = \langle h, e, l, f \rangle \big/ \begin{matrix} h \text{ central} \\ [e,l]=e \quad [l,f]=f \quad [f,e]=h \end{matrix}$

$r = h \otimes l + f \otimes e \qquad R = \exp(r)$

Note $U(g_0)^{\otimes S} = U(\bigoplus_S g_0) = U\left(\langle h_i, e_i, l_i, f_i \rangle \big/ \begin{matrix} h_i \text{ central} \\ [e_i, l_j] = \delta_{ij} e_i \text{ etc.} \end{matrix}\right)$

on to the implementation @ 170120-g0.nb.