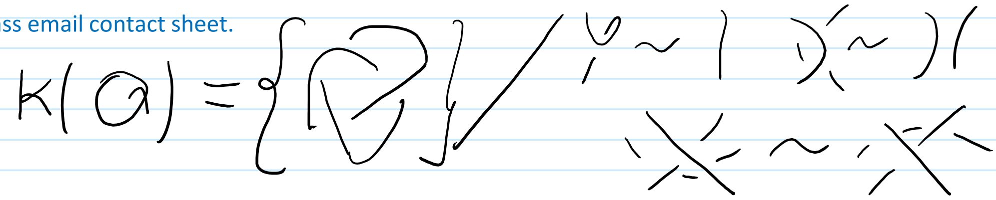
# 17-1350-AKT Fri Jan 13, Brute Hour 1: From Algebras to Invariants

January 8, 2017 9:12 AM

show video?

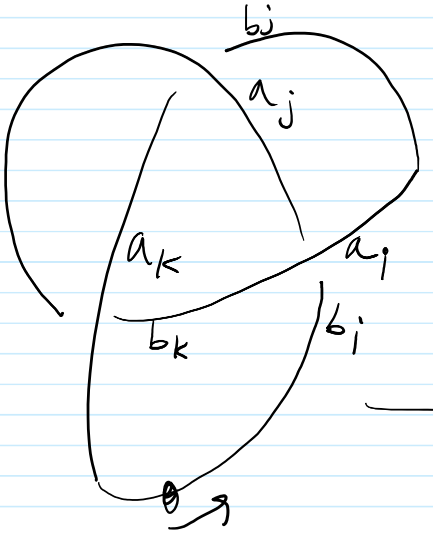
Pass email contact sheet.




$\xrightarrow{Inv}$ something simple

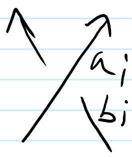
Today: knots, algebras, YBE, CYBE, Lie algebras, universal enveloping algebras, Formulas.

on board

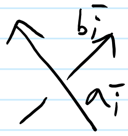



$\longrightarrow \sum_{i,j,k} b_i a_j b_k a_i b_j a_k$

A: unital algebra

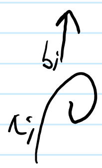



$R^+ \in A \otimes A$; $R^+ = \sum a_i \otimes b_i$ $\quad a_i, b_i \in A$

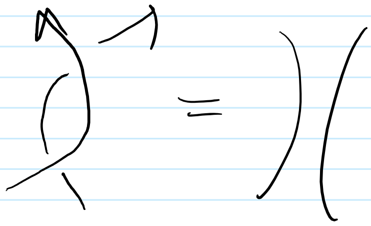



$R^- = \sum a_i^- \otimes b_i^-$

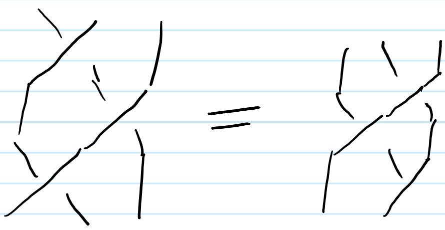
Invariant?




$= |$ $\quad \sum a_i \cdot b_i = 1$ ...



$\Rightarrow R^+ \cdot R^- = 1$



$R^{12}R^{23}R^{13} = R^{13}R^{23}R^{12}$

"YBE"

Idea $R = 1\otimes 1 + hr + h^2 r_2 + \dots$

mod $h^2$: get nothing "CYBE"

mod $\hbar^3$: get $[r^{12}, r^{23}] + [r^{12}, r^{13}] + [r^{23}, r^{13}] = 0$

So maybe $r \in \mathfrak{g} \otimes \mathfrak{g}$, $\mathfrak{g}$ a Lie algebra, and then

$A = U(\mathfrak{g}) = \{\text{words in } \mathfrak{g}\} \big/ xy - yx = [x, y]$

But $U(\mathfrak{g})$ is very hard to work with!

* $\infty$-dim, very complicated formulas.

<u>Sol'n 1</u> use rep. Theory: intrinsically exp-time

<u>Sol'n 2</u> (vdV, BN): restrict to "solvable approx. of SS Lie algebras", and instead of working in $U(\mathfrak{g})$, work in "space of formulas representing elements of $U(\mathfrak{g})$".

Next time: The few Lie-algs we care about, and implementing their UEAs.