17-1350-AKT Fri Feb 17, Brute Hour 6: Computing the g0 invariant

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done

fine

Mathematica at 170210-g0Invariant.nb, then:

Claim under $[e,f]=h$, h central, $\mathbb{O}(e^{tef+\alpha f+\beta e}\,|\,fe) = \mathbb{O}(\nu e^{\nu(tef-\alpha\beta h+\alpha f+\beta e)}\,|\,ef)$,
where $\nu=(1+ht)^{-1}$.

Proofs 4 & 5:

Think: Enough to reorder poly's; hence enough to re-order exps, and this we already know

$$\mathbb{O}(e^{tef+\alpha f+\beta e}/fe) = \mathbb{O}(e^{t\partial_\alpha\partial_\beta}e^{\alpha f+\beta e}/fe) = e^{t\partial_\alpha\partial_\beta}\mathbb{O}(e^{\alpha f+\beta e}/fe)$$

$$= e^{t\partial_\alpha\partial_\beta}\mathbb{O}(e^{-\alpha\beta h+\alpha f+\beta e}/ef) = \mathbb{O}(e^{t\partial_\alpha\partial_\beta}e^{-\alpha\beta h+\alpha f+\beta e}/ef) = \mathbb{O}(\Psi_t/ef)$$

So we are left with a first-year calculus question — compute Ψ_t.

Sol'n 1 Ψ_t satisfies a. $\partial_t\Psi_t = \partial_\alpha\partial_\beta\Psi_t$ b. $\Psi_0 = e^{-\alpha\beta h+\alpha f+\beta e}$

and is determined by these conditions. So it is enough to guess & verify.

Sol'n 2 use (170211) Gaussian pairing:

$$\left\langle \exp\left(\frac{x\subset}{2}+\sum_{i\in I} i\bullet\!\!-\right) \,\middle|\, \exp\left(\frac{\supset y}{2}+\sum_{j\in J} -\!\!\bullet j\right)\right\rangle =$$

$$\exp\left(\log\left(\frac{1}{1-xy}\right)\bigcirc+\sum_{i\in I, j\in J}\frac{i\bullet\!\!-\!\!\bullet j}{1-xy}+\sum_{i_{1,2}\in I}\frac{{}^{i_1}_{i_2}\updownarrow y}{1-xy}+\sum_{j_{1,2}\in J}\frac{x\updownarrow^{j_1}_{j_2}}{1-xy}\right)$$