Pragmatics for the $\mathfrak{g}_l$ Invariant

The Main $\mathfrak{g}_k$ Theorem

Raw Version. The $\mathfrak{g}_k$ invariant of any S-component tangle T can be written in the form $Z(T) = \mathbb{O}\left(\omega\, e^{L+Q+P} \mid \prod_{i\in S} e_i l_i f_i\right)$, where ω is a scalar (meaning, a rational function in the variables h_i and their exponentials $t_i = e^{h_i}$), where $L = \sum a_{ij} h_i l_j$ is a balanced quadratic in the variables h_i and l_j with integer coefficients a_{ij}, where $Q = \sum b_{ij} e_i f_j$ is a balanced quadratic in the variables e_i and f_j with scalar coefficients b_{ij}, and where P is a polynomial in $\{\epsilon, e_i, l_i, f_i\}$ (with scalar coefficients) whose ϵ^d-term is of degree at most $2d+2$ in $\left\{e_i, \sqrt{l_i}, f_i\right\}$. Furthermore, after setting $h_i = h$ and $t_i = t$ for all i, the invariant $Z(T)$ is poly-time computable.

Partial Proof. Indeed,

0. $R^\pm = ?$, $n^\pm = ?$.
1. $\mathbb{O}\left(\mathcal{P}(l, e)\, e^{\gamma l+\beta e} \mid l e\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l+e^\gamma \beta e} \mid e l\right)$,
2. $\mathbb{O}\left(\mathcal{P}(l, f)\, e^{\gamma l+\beta f} \mid f l\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l+e^\gamma \beta f} \mid l f\right)$,
3. $\mathbb{O}\left(\mathcal{P}(e, f)\, e^{\beta e+\alpha f+\delta e f} \mid f e\right) = \mathbb{O}\left(\nu\, \mathcal{P}(\partial_\beta, \partial_\alpha)\, e^{\nu(-\alpha\beta h+\beta e+\alpha f+\delta e f)}\, \Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta) \mid e f\right)$, with $\nu = (1+\mathrm{h}\delta)^{-1}$, and $\Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta)$ as above.

Implementation at $k = l$

$\mathbb{E}$1n[ω, L, Q, P] stands for $\omega\, e^{L+Q}(1+\epsilon\, \mathrm{P})$.

```
DP[x_→D[α_], y_→D[β_]][P_][f_] := Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) ↦ c D[f, {α, m}, {β, n}]]
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Λ[k_][h_, e_, l_, f_, α_, β_, δ_] := Λ[k][h, e, l, f, α, β, δ] = Module[{λ},
    λ = Normal@Series[ℯ^((f α+e β)/(1-α β ϵ)) (1 - α β ϵ)^(-2 l+h/ϵ), {ϵ, 0, k}] /. ℯ- → 1;
    Collect[DP[α→D[f], β→D[e]][λ][ℯ^((f α+e β+e f δ)/(1+h δ))] /. ℯ- → 1, ϵ, Simplify]];
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ϵ /: ϵ^p_ /; p > 1 := 0;
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```
CF[𝔼1n[ω_, L_, Q_, P_]] := 𝔼1n[Together[ω], Together[L], Together[Q], Together[P]];
𝔼1n /: 𝔼1n[ω1_, L1_, Q1_, P1_] 𝔼1n[ω2_, L2_, Q2_, P2_] := CF@𝔼1n[ω1 ω2, L1 + L2, Q1 + Q2, P1 + P2];
𝔼1n[ω1_, L1_, Q1_, P1_] ≡ 𝔼1n[ω2_, L2_, Q2_, P2_] := Simplify[ω1 == ω2 ∧ L1 == L2 ∧ Q1 == Q2 ∧ P1 == P2];
```

0. $R = \mathbb{O}\left(\exp\left(\mathrm{hl} + \frac{e^h-1}{h}\,\mathrm{ef} + P\right) \mid e\otimes\mathrm{lf}\right)$:

```
𝔼1n[X⁺[i_,j_]] := 𝔼1n[1, h[i] l[j], h[i]^-1 (ℯ^h[i] - 1) e[i] f[j], P⁺];
𝔼1n[X⁻[i_,j_]] := 𝔼1n[1, -h[i] l[j], h[i]^-1 (ℯ^-h[i] - 1) e[i] f[j], P⁻];
𝔼1n[p_Times] := 𝔼1n /@ p;
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𝔼1n[X⁺[4,1] X⁺[2,5] X⁺[6,3]]
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$\mathbb{E}1\mathrm{n}\left[1,\ h_4 l_1 + h_6 l_3 + h_2 l_5,\ \frac{1}{h_2 h_4 h_6}\left(-e_6 f_3 h_2 h_4 + e^{h_6} e_6 f_3 h_2 h_4 - e_4 f_1 h_2 h_6 + e^{h_4} e_4 f_1 h_2 h_6 - e_2 f_5 h_4 h_6 + e^{h_2} e_2 f_5 h_4 h_6\right),\ 3\,\mathrm{P}^+\right]$

1. $\mathbb{O}\left(\mathcal{P}(l, e)\, e^{\gamma l+\beta e} \mid l e\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l+e^\gamma \beta e} \mid e l\right)$,
2. $\mathbb{O}\left(\mathcal{P}(l, f)\, e^{\gamma l+\beta f} \mid f l\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l+e^\gamma \beta f} \mid l f\right)$.

```
NO(x:f|e)i_ 1j_[E1n[ω_, L_, Q_, P_]] := With[{q = ℯ^γ β xi + γ 1j},
    CF[E1n[ω, L,
        ℯ^γ β xi + (Q /. xi → 0),
        ℯ^-q DP1j→Dγ,xi→Dβ[P][ℯ^q]
      ] /. {γ → ∂1j L, β → ∂xi Q}]];
```

3. $\mathbb{O}\left(e^{\beta e+\alpha f+\delta ef} \mid fe\right) = \mathbb{O}\left(\nu e^{\nu(-\alpha\beta h+\beta e+\alpha f+\delta ef)} \mid ef\right)$, with $\nu = (1+h\delta)^{-1}$:

```
NOfi_ ej_→k_[E1n[ω_, L_, Q_, P_]] := With[{q = ν (-α β hk + β ek + α fk + δ ek fk)},
    CF[E1n[ν ω, L,
        q + (Q /. fi | ej → 0),
        ℯ^-q DPfi→Dα,ej→Dβ[P][ℯ^q] + (Λ1[hk, ek, lk, fk, α, β, δ] - 1 /. ϵ → 1)
      ] /. ν → (1 + hk δ)^-1 /. {α → ∂fi Q /. ej → 0, β → ∂ej Q /. fi → 0, δ → ∂fi,ej Q}]];
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mi_,j_→k_[Z_] := Module[{x, z}, CF[(Z // NOfi ej→x // NO1i ex // NOfx 1j) /. z_i|j|x → zk]]
```

Meta-associativity and profiling

We did that on March 21. The conclusions were that *m* is indeed meta-associative, and that this can be verified in 400-500 seconds of CPU time, which is pathetic. Profiling told us that almost all execution time was spent within CF.

Pragmatic Simplifications

$\mathbb{E}$1n[ω, L, Q, P] stands for $\omega e^{L+Q}(1+\epsilon P)$; $\mathbb{E}$1p[ω, L, Q, P] stands for $\omega^{-1} e^{L+\omega^{-1}Q}\left(1+\epsilon\,\omega^{-4} P\right)$ /. $e_i \to \frac{t_i-1}{h_i} e_i$, all written in $t_i = e^{h_i}$.

```
E1n[E1p[ω_, L_, Q_, P_]] := CF[PowerExpand /@ CF[E1n[ω^-1, L, ω^-1 Q, ω^-4 P] /. ei_ :→ (ti - 1)/hi ei /. ti_ :→ ℯ^hi]];
CF[E1p[ω_, L_, Q_, P_]] := E1p[Together[ω], Together[L], Together[Q], Together[P]];
E1p[E1n[ω_, L_, Q_, P_]] := CF[E1p[ω^-1, L, ω^-1 Q, ω^-4 P] /. ei_ :→ hi/(ti - 1) ei /. hi_ :→ Log[ti]];
E1p[ω1_, L1_, Q1_, P1_] ≡ E1p[ω2_, L2_, Q2_, P2_] := Simplify[ω1 == ω2 ∧ L1 == L2 ∧ Q1 == Q2 ∧ P1 == P2];
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```
{X⁺1,2 // E1n , X⁺1,2 // E1n // E1p}
```

```
{E1n[1, h1 l2, ((-1 + ℯ^h1) e1 f2)/h1, P⁺], E1p[1, Log[t1] l2, e1 f2, P⁺]}
```

$$\Big\{\zeta n = \mathbb{E}1n\Big[\omega,\ \sum_{i=1}^{4}\sum_{j=1}^{4} a_{i,j}\, h_i\, l_j,\ \sum_{i=1}^{4}\sum_{j=1}^{4} b_{i,j}\, e_i\, f_j,\ a + \sum_{i=1}^{4}\left(a_i\, l_i + b_i\, e_i + c_i\, f_i\right)\Big],$$

$$\zeta p = \mathbb{E}1p\Big[\omega,\ \sum_{i=1}^{4}\sum_{j=1}^{4} a_{i,j}\, \mathrm{Log}[t_i]\, l_j,\ \sum_{i=1}^{4}\sum_{j=1}^{4} b_{i,j}\, e_i\, f_j,\ a + \sum_{i=1}^{4}\left(a_i\, l_i + b_i\, e_i + c_i\, f_i\right)\Big]\Big\}$$

$$\{\mathbb{E}1n[\omega,\ h_1 l_1 a_{1,1} + h_1 l_2 a_{1,2} + h_1 l_3 a_{1,3} + h_1 l_4 a_{1,4} + h_2 l_1 a_{2,1} + h_2 l_2 a_{2,2} + h_2 l_3 a_{2,3} + h_2 l_4 a_{2,4} + h_3 l_1 a_{3,1} + h_3 l_2 a_{3,2} + h_3 l_3 a_{3,3} + h_3 l_4 a_{3,4} + h_4 l_1 a_{4,1} + h_4 l_2 a_{4,2} + h_4 l_3 a_{4,3} + h_4 l_4 a_{4,4},$$
$$e_1 f_1 b_{1,1} + e_1 f_2 b_{1,2} + e_1 f_3 b_{1,3} + e_1 f_4 b_{1,4} + e_2 f_1 b_{2,1} + e_2 f_2 b_{2,2} + e_2 f_3 b_{2,3} + e_2 f_4 b_{2,4} + e_3 f_1 b_{3,1} + e_3 f_2 b_{3,2} + e_3 f_3 b_{3,3} + e_3 f_4 b_{3,4} + e_4 f_1 b_{4,1} + e_4 f_2 b_{4,2} + e_4 f_3 b_{4,3} + e_4 f_4 b_{4,4},$$
$$a + b_1 e_1 + b_2 e_2 + b_3 e_3 + b_4 e_4 + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + a_1 l_1 + a_2 l_2 + a_3 l_3 + a_4 l_4],$$
$$\mathbb{E}1p[\omega,\ \mathrm{Log}[t_1] l_1 a_{1,1} + \mathrm{Log}[t_1] l_2 a_{1,2} + \mathrm{Log}[t_1] l_3 a_{1,3} + \mathrm{Log}[t_1] l_4 a_{1,4} + \mathrm{Log}[t_2] l_1 a_{2,1} + \mathrm{Log}[t_2] l_2 a_{2,2} + \mathrm{Log}[t_2] l_3 a_{2,3} + \mathrm{Log}[t_2] l_4 a_{2,4} + \mathrm{Log}[t_3] l_1 a_{3,1} + \mathrm{Log}[t_3] l_2 a_{3,2} +$$
$$\mathrm{Log}[t_3] l_3 a_{3,3} + \mathrm{Log}[t_3] l_4 a_{3,4} + \mathrm{Log}[t_4] l_1 a_{4,1} + \mathrm{Log}[t_4] l_2 a_{4,2} + \mathrm{Log}[t_4] l_3 a_{4,3} + \mathrm{Log}[t_4] l_4 a_{4,4},$$
$$e_1 f_1 b_{1,1} + e_1 f_2 b_{1,2} + e_1 f_3 b_{1,3} + e_1 f_4 b_{1,4} + e_2 f_1 b_{2,1} + e_2 f_2 b_{2,2} + e_2 f_3 b_{2,3} + e_2 f_4 b_{2,4} + e_3 f_1 b_{3,1} + e_3 f_2 b_{3,2} + e_3 f_3 b_{3,3} + e_3 f_4 b_{3,4} + e_4 f_1 b_{4,1} + e_4 f_2 b_{4,2} + e_4 f_3 b_{4,3} + e_4 f_4 b_{4,4},$$
$$a + b_1 e_1 + b_2 e_2 + b_3 e_3 + b_4 e_4 + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + a_1 l_1 + a_2 l_2 + a_3 l_3 + a_4 l_4]\}$$

Simplify /@ 𝔼1p[ζn]

$$\mathbb{E}1p\Big[\frac{1}{\omega},\ \mathrm{Log}[t_1] l_3 a_{1,3} + \mathrm{Log}[t_1] l_4 a_{1,4} + \mathrm{Log}[t_2] l_3 a_{2,3} + \mathrm{Log}[t_2] l_4 a_{2,4} + \mathrm{Log}[t_3] l_3 a_{3,3} + \mathrm{Log}[t_3] l_4 a_{3,4} + l_1 \left(\mathrm{Log}[t_1] a_{1,1} + \mathrm{Log}[t_2] a_{2,1} + \mathrm{Log}[t_3] a_{3,1} + \mathrm{Log}[t_4] a_{4,1}\right) +$$
$$l_2 \left(\mathrm{Log}[t_1] a_{1,2} + \mathrm{Log}[t_2] a_{2,2} + \mathrm{Log}[t_3] a_{3,2} + \mathrm{Log}[t_4] a_{4,2}\right) + \mathrm{Log}[t_4] l_3 a_{4,3} + \mathrm{Log}[t_4] l_4 a_{4,4},$$
$$\frac{1}{\omega}\left(\frac{\mathrm{Log}[t_1] e_1 \left(f_1 b_{1,1} + f_2 b_{1,2} + f_3 b_{1,3} + f_4 b_{1,4}\right)}{-1+t_1} + \frac{\mathrm{Log}[t_2] e_2 \left(f_1 b_{2,1} + f_2 b_{2,2} + f_3 b_{2,3} + f_4 b_{2,4}\right)}{-1+t_2} + \frac{\mathrm{Log}[t_3] e_3 \left(f_1 b_{3,1} + f_2 b_{3,2} + f_3 b_{3,3} + f_4 b_{3,4}\right)}{-1+t_3} + \frac{\mathrm{Log}[t_4] e_4 \left(f_1 b_{4,1} + f_2 b_{4,2} + f_3 b_{4,3} + f_4 b_{4,4}\right)}{-1+t_4}\right),$$
$$\frac{1}{\omega^4}\left(a + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + a_1 l_1 + a_2 l_2 + a_3 l_3 + a_4 l_4 + \frac{\mathrm{Log}[t_1] b_1 e_1}{-1+t_1} + \frac{\mathrm{Log}[t_2] b_2 e_2}{-1+t_2} + \frac{\mathrm{Log}[t_3] b_3 e_3}{-1+t_3} + \frac{\mathrm{Log}[t_4] b_4 e_4}{-1+t_4}\right)\Big]$$

ζn ≡ (ζn // 𝔼1p // 𝔼1n)

True

ζp ≡ (ζp // 𝔼1n // 𝔼1p)

True

$\mathbb{E}1p[\omega, L, e_1 f_2 + f_1 e_2 + \delta\, e_1 f_1, 0]$ // 𝔼1n

$$\mathbb{E}1n\Big[\frac{1}{\omega},\ L,\ \frac{-e_2 f_1 h_1 + e^{h_2} e_2 f_1 h_1 - \delta e_1 f_1 h_2 + e^{h_1} \delta e_1 f_1 h_2 - e_1 f_2 h_2 + e^{h_1} e_1 f_2 h_2}{\omega h_1 h_2},\ 0\Big]$$

Simplify /@ $\left(\left(\mathbb{E}1p[\omega, L, \omega e_1 \beta + \omega f_1 \alpha + \omega \delta e_1 f_1, 0] \text{ // } \mathbb{E}1n \text{ // } NO_{f_1 e_1 \to 0} \text{ // } \mathbb{E}1p\right) \text{ /. } \{t_0 \to t_1, h_0 \to h_1, l_0 \to l_1\}\right)$

$$\mathbb{E}1p\Big[\omega\left(1-\delta+\delta t_1\right),\ L,\ \omega\left(e_0\left(\beta+\delta f_0\right) + \alpha\left(\beta + f_0 - \beta t_1\right)\right),$$
$$\frac{1}{2\,\mathrm{Log}[t_1]}\omega^4\left(-1+t_1\right)\Big(\alpha^2\beta^2 + 4\alpha\beta\delta + 2\delta^2 - 4\alpha\beta\delta^2 - 4\delta^3 + 2\delta^4 + 4\alpha\beta l_1 + 4\delta l_1 -$$
$$8\alpha\beta\delta l_1 - 12\delta^2 l_1 + 4\alpha\beta\delta^2 l_1 + 12\delta^3 l_1 - 4\delta^4 l_1 - \alpha^2\beta^2 t_1 - 4\alpha\beta\delta t_1 - 2\delta^2 t_1 + 8\alpha\beta\delta^2 t_1 +$$
$$8\delta^3 t_1 - 6\delta^4 t_1 + 8\alpha\beta\delta l_1 t_1 + 12\delta^2 l_1 t_1 - 8\alpha\beta\delta^2 l_1 t_1 - 24\delta^3 l_1 t_1 + 12\delta^4 l_1 t_1 - 4\alpha\beta\delta^2 t_1^2 -$$
$$4\delta^3 t_1^2 + 6\delta^4 t_1^2 + 4\alpha\beta\delta^2 l_1 t_1^2 + 12\delta^3 l_1 t_1^2 - 12\delta^4 l_1 t_1^2 - 2\delta^4 t_1^3 + 4\delta^4 l_1 t_1^3 + \alpha^2\delta f_0^2\left(2-\delta+\delta t_1\right) +$$
$$2\alpha f_0\left(\alpha\beta + 2\delta - 2\delta^2 + 2\delta^2 t_1 + 2\delta l_1\left(1-\delta+\delta t_1\right)^2\right) + \delta e_0^2\left(\beta+\delta f_0\right)\left(\beta\left(2-\delta+\delta t_1\right) + \delta f_0\left(4-3\delta+3\delta t_1\right)\right) +$$
$$2 e_0\Big(\alpha\delta^2 f_0^2\left(3-2\delta+2\delta t_1\right) + \beta\left(\alpha\beta + 2\delta - 2\delta^2 + 2\delta^2 t_1 + 2\delta l_1\left(1-\delta+\delta t_1\right)^2\right) +$$
$$2\delta f_0\left(\delta l_1\left(1-\delta+\delta t_1\right)^2 + \left(2-\delta+\delta t_1\right)\left(\alpha\beta + \delta - \delta^2 + \delta^2 t_1\right)\right)\Big)\Big)\Big]$$

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𝔼1p[ω, L, ω^1 (β e1 + α f1 + δ e1 f1), 0] // 𝔼1n
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$$\mathbb{E}1n\left[\frac{1}{\omega}, L, \frac{-\beta\, e_1 + e^{h_1} \beta\, e_1 - \delta\, e_1 f_1 + e^{h_1} \delta\, e_1 f_1 + \alpha f_1 h_1}{h_1}, 0\right]$$

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Simplify /@ ((𝔼1p[ω, L, ω^1 (β e1 + α f1 + δ e1 f1), 0] // 𝔼1n // NO_{f1 e1→k} // 𝔼1p) /. t1 → tk)
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$$\mathbb{E}1p\Big[\omega \left(1 - \delta + \delta\, t_k\right), L, \omega \left(e_k \left(\beta + \delta f_k\right) + \alpha \left(\beta + f_k - \beta\, t_k\right)\right),$$
$$\frac{1}{2\,\mathrm{Log}[t_k]}\, \omega^4 \left(-1 + t_k\right) \Big(\alpha^2 \beta^2 + 4 \alpha \beta \delta + 2 \delta^2 - 4 \alpha \beta \delta^2 - 4 \delta^3 + 2 \delta^4 + 4 \alpha \beta\, l_k + 4 \delta\, l_k -$$
$$8 \alpha \beta \delta\, l_k - 12 \delta^2 l_k + 4 \alpha \beta \delta^2 l_k + 12 \delta^3 l_k - 4 \delta^4 l_k - \alpha^2 \beta^2 t_k - 4 \alpha \beta \delta\, t_k - 2 \delta^2 t_k + 8 \alpha \beta \delta^2 t_k +$$
$$8 \delta^3 t_k - 6 \delta^4 t_k + 8 \alpha \beta \delta\, l_k t_k + 12 \delta^2 l_k t_k - 8 \alpha \beta \delta^2 l_k t_k - 24 \delta^3 l_k t_k + 12 \delta^4 l_k t_k - 4 \alpha \beta \delta^2 t_k^2 -$$
$$4 \delta^3 t_k^2 + 6 \delta^4 t_k^2 + 4 \alpha \beta \delta^2 l_k t_k^2 + 12 \delta^3 l_k t_k^2 - 12 \delta^4 l_k t_k^2 - 2 \delta^4 t_k^3 + 4 \delta^4 l_k t_k^3 + \alpha^2 \delta f_k^2 \left(2 - \delta + \delta\, t_k\right) +$$
$$2 \alpha f_k \left(\alpha \beta + 2 \delta - 2 \delta^2 + 2 \delta^2 t_k + 2 \delta\, l_k \left(1 - \delta + \delta\, t_k\right)^2\right) + \delta e_k^2 \left(\beta + \delta f_k\right) \left(\beta \left(2 - \delta + \delta\, t_k\right) + \delta f_k \left(4 - 3 \delta + 3 \delta\, t_k\right)\right) +$$
$$2 e_k \Big(\alpha \delta^2 f_k^2 \left(3 - 2 \delta + 2 \delta\, t_k\right) + \beta \left(\alpha \beta + 2 \delta - 2 \delta^2 + 2 \delta^2 t_k + 2 \delta\, l_k \left(1 - \delta + \delta\, t_k\right)^2\right) +$$
$$2 \delta f_k \left(\delta\, l_k \left(1 - \delta + \delta\, t_k\right)^2 + \left(2 - \delta + \delta\, t_k\right) \left(\alpha \beta + \delta - \delta^2 + \delta^2 t_k\right)\right)\Big)\Big)\Big]$$

```
𝔼1p /: 𝔼1p[ω1_, L1_, Q1_, P1_] 𝔼1p[ω2_, L2_, Q2_, P2_] :=
  CF@𝔼1p[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

```
NO_{f_i_ e_j_→k_}[𝔼1p[ω_, L_, Q_, P_]] := Module[{α, β, δ, μ, q, Λ},
   q = ((1 - t_k) α β + β e_k + δ e_k f_k + α f_k) / μ;
   Λ = 1/(2 Log[t_k]) ω^4 (-1 + t_k)
     (α^2 β^2 + 4 α β δ + 2 δ^2 - 4 α β δ^2 - 4 δ^3 + 2 δ^4 + 4 α β l_k + 4 δ l_k - 8 α β δ l_k - 12 δ^2 l_k + 4 α β δ^2 l_k + 12 δ^3 l_k -
       4 δ^4 l_k - α^2 β^2 t_k - 4 α β δ t_k - 2 δ^2 t_k + 8 α β δ^2 t_k + 8 δ^3 t_k - 6 δ^4 t_k + 8 α β δ l_k t_k + 12 δ^2 l_k t_k -
       8 α β δ^2 l_k t_k - 24 δ^3 l_k t_k + 12 δ^4 l_k t_k - 4 α β δ^2 t_k^2 - 4 δ^3 t_k^2 + 6 δ^4 t_k^2 + 4 α β δ^2 l_k t_k^2 + 12 δ^3 l_k t_k^2 - 12 δ^4 l_k t_k^2 -
       2 δ^4 t_k^3 + 4 δ^4 l_k t_k^3 + α^2 δ f_k^2 (2 - δ + δ t_k) + 2 α f_k (α β + 2 δ - 2 δ^2 + 2 δ^2 t_k + 2 δ l_k (1 - δ + δ t_k)^2) +
       δ e_k^2 (β + δ f_k) (β (2 - δ + δ t_k) + δ f_k (4 - 3 δ + 3 δ t_k)) +
       2 e_k (α δ^2 f_k^2 (3 - 2 δ + 2 δ t_k) + β (α β + 2 δ - 2 δ^2 + 2 δ^2 t_k + 2 δ l_k (1 - δ + δ t_k)^2) +
          2 δ f_k (δ l_k (1 - δ + δ t_k)^2 + (2 - δ + δ t_k) (α β + δ - δ^2 + δ^2 t_k))));
   CF[
    𝔼1p[μ ω, L, μ ω q + μ (Q /. f_i | e_j → 0), μ^4 (DP_{f_i→D_α, e_j→D_β}[P][𝕖^q] /. 𝕖- → 1) + ω^4 Λ[k]] /. μ → 1 + (t_k - 1) δ /.
     {α → ω^-1 (∂_{f_i} Q /. e_j → 0), β → ω^-1 (∂_{e_j} Q /. f_i → 0), δ → ω^-1 ∂_{f_i,e_j} Q}
   ]];
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```
NO_{l_j_ (x:e|f)_i_→k_}[𝔼1p[ω_, L_, Q_, P_]] := With[{q = 𝕖^γ β x_k + γ l_k}, CF[
    𝔼1p[ω, γ l_k + (L /. l_j → 0), ω 𝕖^γ β x_k + (Q /. x_i → 0), 𝕖^-q DP_{l_j→D_γ, x_i→D_β}[P][𝕖^q]] /. {γ → ∂_{l_j} L, β → ω^-1 ∂_{x_i} Q}]];
```

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𝔤n
```

$$\mathbb{E}1n\big[\omega, h_1 l_1 a_{1,1} + h_1 l_2 a_{1,2} + h_1 l_3 a_{1,3} + h_1 l_4 a_{1,4} + h_2 l_1 a_{2,1} + h_2 l_2 a_{2,2} + h_2 l_3 a_{2,3} +$$
$$h_2 l_4 a_{2,4} + h_3 l_1 a_{3,1} + h_3 l_2 a_{3,2} + h_3 l_3 a_{3,3} + h_3 l_4 a_{3,4} + h_4 l_1 a_{4,1} + h_4 l_2 a_{4,2} + h_4 l_3 a_{4,3} + h_4 l_4 a_{4,4},$$
$$e_1 f_1 b_{1,1} + e_1 f_2 b_{1,2} + e_1 f_3 b_{1,3} + e_1 f_4 b_{1,4} + e_2 f_1 b_{2,1} + e_2 f_2 b_{2,2} + e_2 f_3 b_{2,3} + e_2 f_4 b_{2,4} +$$
$$e_3 f_1 b_{3,1} + e_3 f_2 b_{3,2} + e_3 f_3 b_{3,3} + e_3 f_4 b_{3,4} + e_4 f_1 b_{4,1} + e_4 f_2 b_{4,2} + e_4 f_3 b_{4,3} + e_4 f_4 b_{4,4},$$
$$a + b_1 e_1 + b_2 e_2 + b_3 e_3 + b_4 e_4 + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + a_1 l_1 + a_2 l_2 + a_3 l_3 + a_4 l_4\big]$$

lhs = ζn // $\text{NO}_{f_1\,e_1\to 5}$ // 𝔼1p

$\mathbb{E}1\text{p}\Big[\frac{1+\text{Log}[t_5]\,b_{1,1}}{\omega}$, $\text{Log}[t_1]\,l_1\,a_{1,1}+\text{Log}[t_1]\,l_2\,a_{1,2}+\text{Log}[t_1]\,l_3\,a_{1,3}+$ ⋯11⋯ $+\text{Log}[t_4]\,l_3\,a_{4,3}+\text{Log}[t_4]\,l_4\,a_{4,4}$,

$\frac{\cdots 1\cdots}{\cdots 1\cdots}$, $\frac{\cdots 22895\cdots+2\cdots 8\cdots\, b_4^2 \cdots 1\cdots \cdots 1\cdots - \text{Log}[t_4]^2\,\text{Log}[t_5]\,e_4^2\,f_4^2\,t_2^2\,t_3^2\,t_5^2\,b_{1,4}^2\,b_{4,1}^2}{2\,\omega^4\,(-1+t_2)^2\,(-1+t_3)^2\,(-1+t_4)^2\,(-1+t_5)^2}\Big]$

large output | show less | show more | show all | set size limit...

rhs = (ζn // 𝔼1p // $\text{NO}_{f_1\,e_1\to 5}$)

$\mathbb{E}1\text{p}\Big[\frac{-1+t_1-\text{Log}[t_1]\,b_{1,1}+\text{Log}[t_1]\,t_5\,b_{1,1}}{\omega\,(-1+t_1)}$,

$\text{Log}[t_1]\,l_1\,a_{1,1}+\text{Log}[t_1]\,l_2\,a_{1,2}+\text{Log}[t_1]\,l_3\,a_{1,3}+$ ⋯11⋯ $+\text{Log}[t_4]\,l_3\,a_{4,3}+\text{Log}[t_4]\,l_4\,a_{4,4}$,

$\frac{\cdots 1\cdots}{\cdots 1\cdots}$, $\frac{-a+\text{Log}[t_2]\,b_2\,e_2+\cdots 6266\cdots+t_1^4\,t_2\,t_3\,t_4\,\frac{(-1+t_5)\,(\cdots 1\cdots)}{2\,\omega^4\,\text{Log}[t_5]}[5]}{\omega^4\,(-1+t_1)^4\,(-1+t_2)\,(-1+t_3)\,(-1+t_4)}\Big]$

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Simplify[lhs⟦1⟧ == rhs⟦1⟧]

$$\frac{\left(\text{Log}[t_1]-\text{Log}[t_5]+\text{Log}[t_5]\,t_1-\text{Log}[t_1]\,t_5\right)b_{1,1}}{\omega\,(-1+t_1)} == 0$$