

$$M = \begin{pmatrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi \end{pmatrix};$$

$$\{y_1, y_2, y_3\} == M \cdot \{x_1, x_2, x_3\}$$

$$\{y_1, y_2, y_3\} == \{\alpha x_1 + \beta x_2 + \theta x_3, \gamma x_1 + \delta x_2 + \epsilon x_3, \phi x_1 + \psi x_2 + \Xi x_3\}$$

$$\text{eqns} = \text{Thread}[\{y_1, y_2, y_3\} == M \cdot \{x_1, x_2, x_3\}] \cup \{y_1 == x_2\}$$

$$\{y_1 == x_2, y_1 == \alpha x_1 + \beta x_2 + \theta x_3, y_2 == \gamma x_1 + \delta x_2 + \epsilon x_3, y_3 == \phi x_1 + \psi x_2 + \Xi x_3\}$$

$$\text{Solve}[\text{eqns}, \{y_2, y_3\}]$$

{}

$$\{\text{sol}\} = \text{Solve}[\text{eqns}, \{x_2, y_1, y_2, y_3\}]$$

$$\left\{ \left\{ x_2 \rightarrow -\frac{\alpha x_1 + \theta x_3}{-1 + \beta}, y_1 \rightarrow -\frac{\alpha x_1 + \theta x_3}{-1 + \beta}, \right. \right. \\ \left. \left. y_2 \rightarrow -\frac{\gamma x_1 - \beta \gamma x_1 + \alpha \delta x_1 + \epsilon x_3 - \beta \epsilon x_3 + \delta \theta x_3}{-1 + \beta}, y_3 \rightarrow -\frac{\phi x_1 - \beta \phi x_1 + \alpha \psi x_1 + \Xi x_3 - \beta \Xi x_3 + \theta \psi x_3}{-1 + \beta} \right\} \right\}$$

$$\text{Table}[\text{Coefficient}[y_i /. \text{sol}, x_j] // \text{Apart}, \{\mathbf{i}, \{2, 3\}\}, \{\mathbf{j}, \{1, 3\}\}] // \text{MatrixForm}$$

$$\begin{pmatrix} \gamma - \frac{\alpha \delta}{-1 + \beta} & \epsilon - \frac{\delta \theta}{-1 + \beta} \\ \phi - \frac{\alpha \psi}{-1 + \beta} & \Xi - \frac{\theta \psi}{-1 + \beta} \end{pmatrix}$$