```
R_{n_,i_,j_} := ReplacePart[IdentityMatrix[n], {{i, i} → α, {i, j} → β, {j, i} → γ, {j, j} → δ}];
```

```
R_{2,1,2} // MatrixForm
```

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

```
R_{3,1,3} // MatrixForm
```

$$\begin{pmatrix} \alpha & 0 & \beta \\ 0 & 1 & 0 \\ \gamma & 0 & \delta \end{pmatrix}$$

```
R_{3,1,2}.R_{3,1,3}.R_{3,2,3}
```

$$\left\{\left\{\alpha^2,\ \alpha\,\beta+\alpha\,\beta\,\gamma,\ \beta^2+\alpha\,\beta\,\delta\right\},\ \left\{\alpha\,\gamma,\ \beta\,\gamma^2+\alpha\,\delta,\ \beta\,\delta+\beta\,\gamma\,\delta\right\},\ \left\{\gamma,\ \gamma\,\delta,\ \delta^2\right\}\right\}$$

```
eqns = Thread[Flatten[R_{3,1,2}.R_{3,1,3}.R_{3,2,3}] == Flatten[R_{3,2,3}.R_{3,1,3}.R_{3,1,2}]]
```

$$\left\{\text{True},\ \alpha\,\beta+\alpha\,\beta\,\gamma==\alpha\,\beta,\ \beta^2+\alpha\,\beta\,\delta==\beta,\ \alpha\,\gamma==\alpha\,\gamma+\alpha\,\beta\,\gamma,\right.$$
$$\left.\beta\,\gamma^2+\alpha\,\delta==\beta^2\,\gamma+\alpha\,\delta,\ \beta\,\delta+\beta\,\gamma\,\delta==\beta\,\delta,\ \gamma==\gamma^2+\alpha\,\gamma\,\delta,\ \gamma\,\delta==\gamma\,\delta+\beta\,\gamma\,\delta,\ \text{True}\right\}$$

```
Solve[eqns, {α, β, γ, δ}]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\{\{\beta\to 0,\ \gamma\to 0\},\ \{\beta\to 0,\ \gamma\to 1-\alpha\,\delta\},\ \{\beta\to 1-\alpha\,\delta,\ \gamma\to 0\},\ \{\alpha\to 0,\ \beta\to 1,\ \gamma\to 1,\ \delta\to 0\}\}$$

```
MatrixForm /@ (R_{2,1,2} /. Solve[eqns, {α, β, γ, δ}])
```

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{\begin{pmatrix} \alpha & 0 \\ 0 & \delta \end{pmatrix},\ \begin{pmatrix} \alpha & 0 \\ 1-\alpha\,\delta & \delta \end{pmatrix},\ \begin{pmatrix} \alpha & 1-\alpha\,\delta \\ 0 & \delta \end{pmatrix},\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\}$$

```
MatrixForm /@ (R_{2,1,2} /. Solve[eqns ⋃ {α == 0, δ == t}, {α, β, γ, δ}])
```

$$\left\{\begin{pmatrix} 0 & 0 \\ 1 & t \end{pmatrix},\ \begin{pmatrix} 0 & 1 \\ 0 & t \end{pmatrix},\ \begin{pmatrix} 0 & 0 \\ 0 & t \end{pmatrix}\right\}$$

```
MatrixForm /@ (R_{2,1,2} /. Solve[eqns ⋃ {α == 1, δ == t}, {α, β, γ, δ}])
```

$$\left\{\begin{pmatrix} 1 & 0 \\ 1-t & t \end{pmatrix},\ \begin{pmatrix} 1 & 1-t \\ 0 & t \end{pmatrix},\ \begin{pmatrix} 1 & 0 \\ 0 & t \end{pmatrix}\right\}$$