

Pensieve header: Computing the Jones polynomial, further details and a faster procedure.

## The Jones Polynomial of the Trefoil Knot

Jones:  $\nearrow \mapsto q$   $(-q^2 \searrow)$   $\bigcirc \mapsto q + q^{-1}$   
 $\nearrow \mapsto -q^{-2} \searrow + q^{-1}$

<< KnotTheory`

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.  
 Read more at <http://katlas.org/wiki/KnotTheory>.

**K = Knot**[3, 1]

Knot[3, 1]

**PD**[K]

**KnotTheory**: Loading precomputed data in PD4Knots`.

PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]]

**t1 = PD**[K] /. **x\_X** => **If**[**PositiveQ**[x], **Xp**@@x, **Xm**@@x]

PD[Xm[1, 4, 2, 5], Xm[3, 6, 4, 1], Xm[5, 2, 6, 3]]

**t2 = Times**@@**t1**

Xm[1, 4, 2, 5] Xm[3, 6, 4, 1] Xm[5, 2, 6, 3]

**t3 = t2** /. {

- Xp**[i\_, j\_, k\_, l\_] => **q** \* **P**[i, j] **P**[k, l] - **q**<sup>2</sup> \* **P**[i, l] **P**[j, k],
- Xm**[i\_, j\_, k\_, l\_] => -**q**<sup>-2</sup> \* **P**[i, j] **P**[k, l] + **1/q** \* **P**[i, l] **P**[j, k]

$$\left( \frac{P[1, 5] P[2, 4]}{q} - \frac{P[1, 4] P[2, 5]}{q^2} \right) \left( \frac{P[2, 6] P[3, 5]}{q} - \frac{P[2, 5] P[3, 6]}{q^2} \right) \left( -\frac{P[1, 4] P[3, 6]}{q^2} + \frac{P[1, 3] P[4, 6]}{q} \right)$$

**t4 = Expand[t3]**

$$\begin{aligned}
 & -\frac{1}{q^4} P[1, 4] P[1, 5] P[2, 4] P[2, 6] P[3, 5] P[3, 6] + \frac{1}{q^5} \\
 & P[1, 4]^2 P[2, 5] P[2, 6] P[3, 5] P[3, 6] + \frac{1}{q^5} P[1, 4] P[1, 5] P[2, 4] P[2, 5] P[3, 6]^2 - \\
 & \frac{1}{q^6} P[1, 4]^2 P[2, 5]^2 P[3, 6]^2 + \frac{1}{q^3} P[1, 3] P[1, 5] P[2, 4] P[2, 6] P[3, 5] P[4, 6] - \\
 & \frac{1}{q^4} P[1, 3] P[1, 4] P[2, 5] P[2, 6] P[3, 5] P[4, 6] - \frac{1}{q^4} \\
 & P[1, 3] P[1, 5] P[2, 4] P[2, 5] P[3, 6] P[4, 6] + \frac{1}{q^5} P[1, 3] P[1, 4] P[2, 5]^2 P[3, 6] P[4, 6]
 \end{aligned}$$

**SetAttributes[P, Orderless]**

**t5 = t4 /. P[a\_, b\_] P[b\_, c\_] => P[a, c]**

$$\begin{aligned}
 & -\frac{1}{q^6} P[1, 4]^2 P[2, 5]^2 P[3, 6]^2 + \frac{P[3, 6]^2 P[4, 5]^2}{q^5} + \\
 & \frac{P[2, 5]^2 P[4, 6]^2}{q^5} + \frac{P[3, 5]^2 P[4, 6]^2}{q^3} - \frac{3 P[5, 6]^2}{q^4} + \frac{P[1, 4]^2 P[5, 6]^2}{q^5}
 \end{aligned}$$

**t6 = t5 /. {**  
 $P[i_, i_] \Rightarrow (q + 1/q),$   
 $P[i_, j_]^2 \Rightarrow (q + 1/q)$   
**}**

$$-\frac{3 \left(\frac{1}{q} + q\right)}{q^4} + \frac{3 \left(\frac{1}{q} + q\right)^2}{q^5} + \frac{\left(\frac{1}{q} + q\right)^2}{q^3} - \frac{\left(\frac{1}{q} + q\right)^3}{q^6}$$

**Expand[t6]**

$$-\frac{1}{q^9} + \frac{1}{q^5} + \frac{1}{q^3} + \frac{1}{q}$$

**Simplify[t6]**

$$\frac{-1 + q^4 + q^6 + q^8}{q^9}$$

**Jones[Knot[3, 1]][q]**

**KnotTheory:** Loading precomputed data in Jones4Knots`.

$$-\frac{1}{q^4} + \frac{1}{q^3} + \frac{1}{q}$$

**Simplify**  $\left[ \frac{t6}{q + q^{-1}} \right]$  /.  $q \rightarrow \sqrt{q}$

$$\frac{-1 + q + q^3}{q^4}$$


---

## A Jones Polynomial Program

```
J[K_] := Module[{t1, t2, t3, t4, t5, t6, P},
  SetAttributes[P, Orderless];
  t1 = PD[K] /. x_X => If[PositiveQ[x], Xp@@x, Xm@@x];
  t2 = Times@@t1;
  t3 = t2 /. {
    Xp[i_, j_, k_, l_] => q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
    Xm[i_, j_, k_, l_] => -q^(-2) * P[i, j] P[k, l] + 1/q * P[i, l] P[j, k]
  };
  t4 = Expand[t3];
  t5 = t4 //. P[a_, b_] P[b_, c_] => P[a, c];
  t6 = t5 /. {
    P[i_, i_] => (q + 1/q),
    P[i_, j_]^2 => (q + 1/q)
  };
  Simplify[ $\frac{t6}{q + q^{-1}}$  /. q ->  $\sqrt{q}$ ]
]
```

J[Knot[8, 17]]

$$7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$$

Jones[Knot[8, 17]][q]

$$7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$$

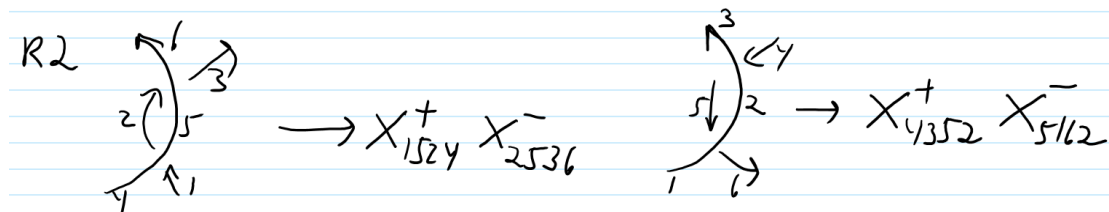


**Xp**[2, 2, 3, 1] // **TJ**

P[1, 3]

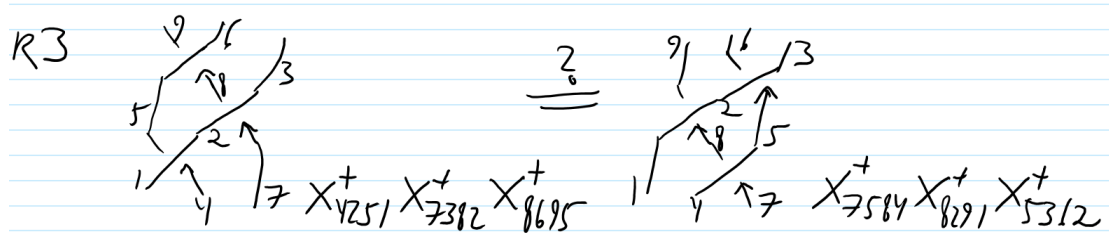
**Xm**[1, 2, 2, 3] // **TJ**

P[1, 3]



**TJ** /@ {**Xp**[1, 5, 2, 4] **Xm**[2, 5, 3, 6], **Xp**[4, 3, 5, 2] **Xm**[5, 1, 6, 2]}

{P[1, 3] P[4, 6], P[1, 3] P[4, 6]}



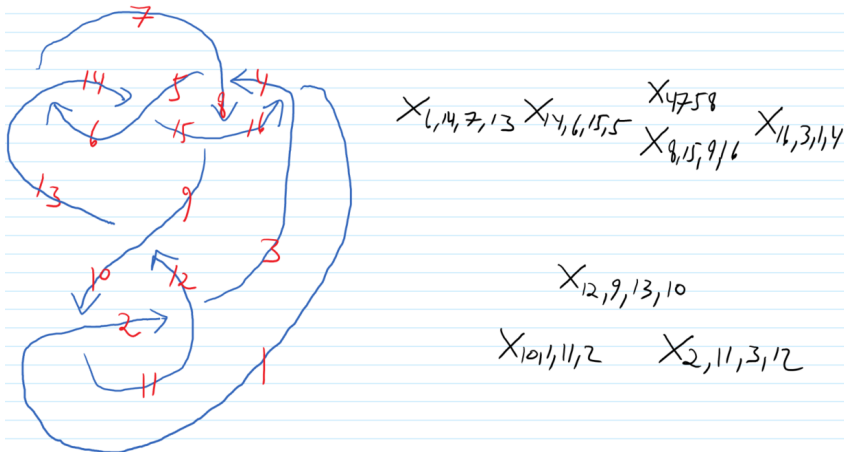
**TJ** /@ {**Xp**[4, 2, 5, 1] **Xp**[7, 3, 8, 2] **Xp**[8, 6, 9, 5], **Xp**[7, 5, 8, 4] **Xp**[8, 2, 9, 1] **Xp**[5, 3, 6, 2]}

$$\{q^3 (P[1, 9] (P[3, 7] P[4, 6] - q P[3, 6] P[4, 7]) + q (q P[1, 3] P[4, 7] P[6, 9] + P[1, 4] (-P[3, 7] P[6, 9] + q P[3, 6] P[7, 9]))) , q^3 (P[1, 9] (P[3, 7] P[4, 6] - q P[3, 6] P[4, 7]) + q (q P[1, 3] P[4, 7] P[6, 9] + P[1, 4] (-P[3, 7] P[6, 9] + q P[3, 6] P[7, 9])))\}$$

%[[1] == %[[2]]

True

## Analyzing a knot suggested by David Vincent



$K1 = PD[X[6, 14, 7, 13], X[14, 6, 15, 5], X[4, 7, 5, 8], X[8, 15, 9, 16], X[16, 3, 1, 4], X[12, 9, 13, 10], X[10, 1, 11, 2], X[2, 11, 3, 12]];$

$J1 = J[K1]$

$$\frac{1}{q^7} (1 - 3q + 4q^2 - 5q^3 + 6q^4 - 5q^5 + 4q^6 - 2q^7 + q^8)$$

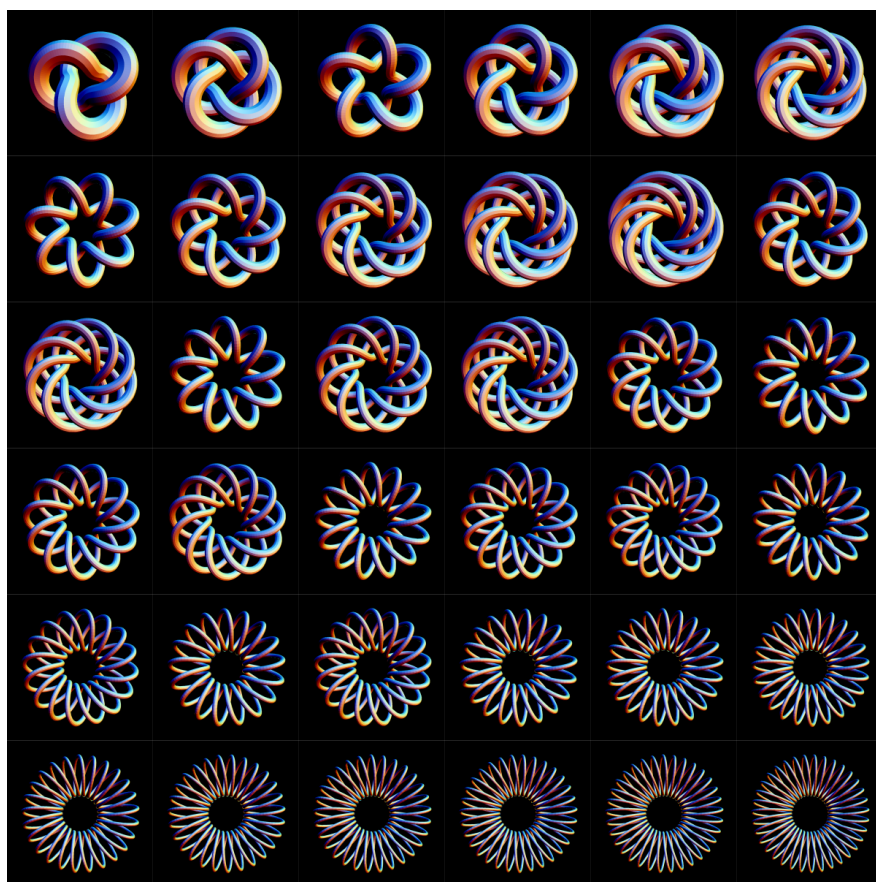
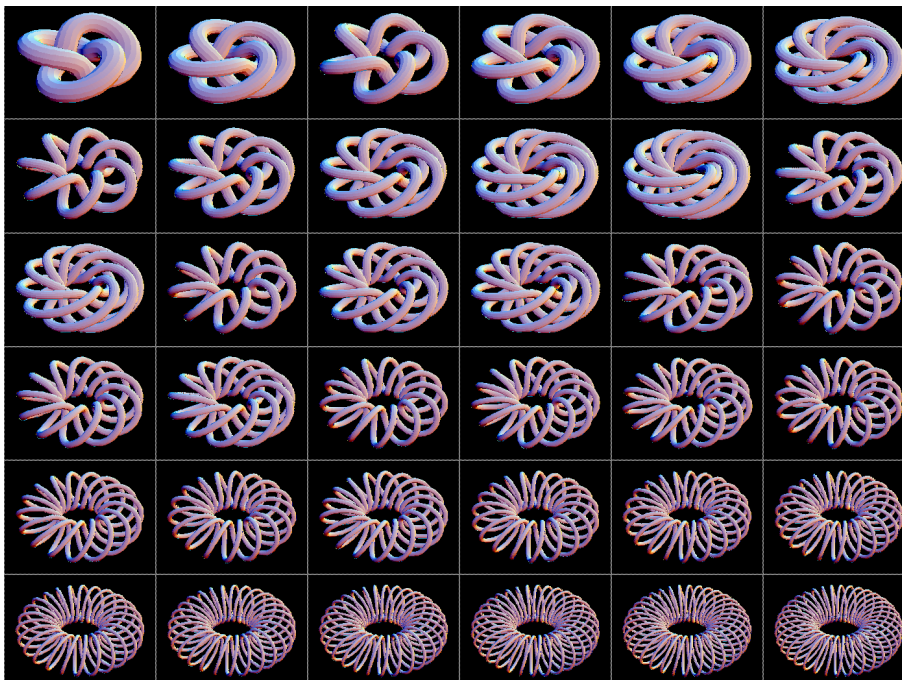
$Select[AllKnots[{3, 8}], J[\#] == J1 \&]$

$\{Knot[8, 14]\}$

## A Faster Jones Program

```
FJ[K_] := Module[{todo, touched = {}, x, J = 1},
  SetAttributes[P, Orderless];
  todo = List@@PD[K] /. x_X => If[PositiveQ[x], Xp@@x, Xm@@x];
  While[Length[todo] > 0,
    x = First@MaximalBy[todo, Length[touched \cap (List@@#)] &];
    J *= x /. {
      Xp[i_, j_, k_, L_] => q * P[i, j] P[k, L] - q^2 * P[i, L] P[j, k],
      Xm[i_, j_, k_, L_] => -q^(-2) * P[i, j] P[k, L] + 1/q * P[i, L] P[j, k]
    };
    J = Expand[J] //. {
      P[a_, b_] P[b_, c_] => P[a, c] /. {
        P[i_, i_] => (q + 1/q),
        P[i_, j_]^2 => (q + 1/q)
      };
    todo = DeleteCases[todo, x];
  ];
  Simplify[ $\frac{J}{q + q^{-1}}$  /. q ->  $\sqrt{q}$ ]
]
```





? TorusKnots



TorusKnots[n\_] returns a list of all torus knots with up to n crossings.

### TorusKnots [100]

```
{TorusKnot [3, 2], TorusKnot [5, 2], TorusKnot [7, 2], TorusKnot [4, 3], TorusKnot [9, 2],
TorusKnot [5, 3], TorusKnot [11, 2], TorusKnot [13, 2], TorusKnot [7, 3], TorusKnot [5, 4],
TorusKnot [15, 2], TorusKnot [8, 3], TorusKnot [17, 2], TorusKnot [19, 2], TorusKnot [10, 3],
TorusKnot [7, 4], TorusKnot [21, 2], TorusKnot [11, 3], TorusKnot [23, 2], TorusKnot [6, 5],
TorusKnot [25, 2], TorusKnot [13, 3], TorusKnot [9, 4], TorusKnot [27, 2], TorusKnot [7, 5],
TorusKnot [14, 3], TorusKnot [29, 2], TorusKnot [31, 2], TorusKnot [8, 5], TorusKnot [16, 3],
TorusKnot [11, 4], TorusKnot [33, 2], TorusKnot [17, 3], TorusKnot [7, 6], TorusKnot [35, 2],
TorusKnot [9, 5], TorusKnot [37, 2], TorusKnot [19, 3], TorusKnot [13, 4], TorusKnot [39, 2],
TorusKnot [20, 3], TorusKnot [41, 2], TorusKnot [43, 2], TorusKnot [11, 5], TorusKnot [22, 3],
TorusKnot [15, 4], TorusKnot [45, 2], TorusKnot [23, 3], TorusKnot [47, 2], TorusKnot [8, 7],
TorusKnot [12, 5], TorusKnot [49, 2], TorusKnot [25, 3], TorusKnot [17, 4], TorusKnot [51, 2],
TorusKnot [13, 5], TorusKnot [26, 3], TorusKnot [53, 2], TorusKnot [9, 7], TorusKnot [11, 6],
TorusKnot [55, 2], TorusKnot [14, 5], TorusKnot [28, 3], TorusKnot [19, 4], TorusKnot [57, 2],
TorusKnot [29, 3], TorusKnot [59, 2], TorusKnot [10, 7], TorusKnot [61, 2], TorusKnot [31, 3],
TorusKnot [9, 8], TorusKnot [21, 4], TorusKnot [63, 2], TorusKnot [16, 5], TorusKnot [32, 3],
TorusKnot [13, 6], TorusKnot [65, 2], TorusKnot [11, 7], TorusKnot [67, 2], TorusKnot [17, 5],
TorusKnot [34, 3], TorusKnot [23, 4], TorusKnot [69, 2], TorusKnot [35, 3], TorusKnot [71, 2],
TorusKnot [12, 7], TorusKnot [18, 5], TorusKnot [73, 2], TorusKnot [37, 3], TorusKnot [25, 4],
TorusKnot [75, 2], TorusKnot [19, 5], TorusKnot [38, 3], TorusKnot [11, 8], TorusKnot [77, 2],
TorusKnot [13, 7], TorusKnot [79, 2], TorusKnot [10, 9], TorusKnot [40, 3], TorusKnot [27, 4],
TorusKnot [81, 2], TorusKnot [41, 3], TorusKnot [83, 2], TorusKnot [21, 5], TorusKnot [17, 6],
TorusKnot [85, 2], TorusKnot [43, 3], TorusKnot [29, 4], TorusKnot [87, 2], TorusKnot [11, 9],
TorusKnot [22, 5], TorusKnot [44, 3], TorusKnot [89, 2], TorusKnot [15, 7], TorusKnot [13, 8],
TorusKnot [91, 2], TorusKnot [23, 5], TorusKnot [46, 3], TorusKnot [31, 4],
TorusKnot [93, 2], TorusKnot [47, 3], TorusKnot [19, 6], TorusKnot [95, 2],
TorusKnot [16, 7], TorusKnot [24, 5], TorusKnot [97, 2], TorusKnot [49, 3],
TorusKnot [11, 10], TorusKnot [33, 4], TorusKnot [99, 2], TorusKnot [50, 3]}
```

```
LaunchKernels[];
DistributeDefinitions /@ {"KnotTheory`, "Global`"}
```

**KnotTheory**: The minimum braids representing the knots with up to 10 crossings were provided by Thomas Gittings. See arXiv:math.GT/0401051.

... **Function**: Slot number 2 in  
Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] & cannot be filled from (Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] &)[KernelObject[1, local]].

... **Function**: Slot number 2 in  
Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] & cannot be filled from (Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] &)[KernelObject[1, local]].

... **Function**: Slot number 2 in  
Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] & cannot be filled from (Parallel`Kernels`Private`sendCatch[#, Parallel`Client`makeDefinitions[Language`DefinitionList[<<1>>]]] &)[KernelObject[1, local]].

... **General**: Further output of Function::slotn will be suppressed during this calculation.

```
{{KnotTheory`}, {Global`}}
```

```
WaitAll[{
  ParallelSubmit[
    t0 = TimeUsed[];
    Do[
      {t1, J1} = Timing@J[K];
      Print["Slow Jones: ", {K, Length@PD[K], TimeUsed[] - t0, t1, J1}],
      {K, TorusKnots[100]}
    ]
  ],
  ParallelSubmit[
    t0 = TimeUsed[];
    Do[
      {t1, J1} = Timing@FJ[K];
      Print["Fast Jones: ", {K, Length@PD[K], TimeUsed[] - t0, t1, J1}],
      {K, TorusKnots[100]}
    ]
  ]
}]
```

Fast Jones: {KnotTheory`TorusKnot[3, 2], 3, 0.016, 0.015625,  $q + q^3 - q^4$ }

Slow Jones: {KnotTheory`TorusKnot[3, 2], 3, 0.016, 0.,  $q + q^3 - q^4$ }

Fast Jones: {KnotTheory`TorusKnot[5, 2], 5, 0.016, 0.,  $q^2 + q^4 - q^5 + q^6 - q^7$ }

Slow Jones: {KnotTheory`TorusKnot[5, 2], 5, 0.031, 0.015625,  $q^2 + q^4 - q^5 + q^6 - q^7$ }

Fast Jones: {KnotTheory`TorusKnot[7, 2], 7, 0.031, 0.015625,  $q^3 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10}$ }

Slow Jones: {KnotTheory`TorusKnot[7, 2], 7, 0.078, 0.046875,  $q^3 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10}$ }

Fast Jones: {KnotTheory`TorusKnot[4, 3], 8, 0.047, 0.,  $q^3 + q^5 - q^8$ }

Slow Jones: {KnotTheory`TorusKnot [4, 3], 8, 0.156, 0.078125,  $q^3 + q^5 - q^8$ }

Fast Jones: {KnotTheory`TorusKnot [9, 2], 9, 0.047, 0.,  $q^4 (1 + q^2 - q^3 + q^4 - q^5 + q^6 - q^7 + q^8 - q^9)$ }

Fast Jones: {KnotTheory`TorusKnot [5, 3], 10, 0.078, 0.03125,  $q^4 + q^6 - q^{10}$ }

Fast Jones: {KnotTheory`TorusKnot [11, 2], 11, 0.094, 0.015625,  $-q^5 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11})$ }

Fast Jones: {KnotTheory`TorusKnot [13, 2], 13, 0.125, 0.03125,  $-q^6 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13})$ }

Fast Jones: {KnotTheory`TorusKnot [7, 3], 14, 0.156, 0.03125,  $q^6 + q^8 - q^{14}$ }

Fast Jones: {KnotTheory`TorusKnot [5, 4], 15, 0.235, 0.078125,  $q^6 (1 + q^2 + q^4 - q^5 - q^7)$ }

Fast Jones: {KnotTheory`TorusKnot [15, 2], 15, 0.266, 0.03125,  $-q^7 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15})$ }

Fast Jones: {KnotTheory`TorusKnot [8, 3], 16, 0.313, 0.046875,  $q^7 + q^9 - q^{16}$ }

Fast Jones: {KnotTheory`TorusKnot [17, 2], 17, 0.36, 0.03125,  $-q^8 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17})$ }

Fast Jones: {KnotTheory`TorusKnot [19, 2], 19, 0.391, 0.03125,  $-q^9 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19})$ }

Slow Jones: {KnotTheory`TorusKnot [9, 2], 9, 0.437, 0.28125,  $q^4 (1 + q^2 - q^3 + q^4 - q^5 + q^6 - q^7 + q^8 - q^9)$ }

Fast Jones: {KnotTheory`TorusKnot [10, 3], 20, 0.469, 0.078125,  $q^9 + q^{11} - q^{20}$ }

Fast Jones: {KnotTheory`TorusKnot [7, 4], 21, 0.61, 0.140625,  $-q^9 (-1 - q^2 - q^4 + q^5 - q^6 + q^7 + q^9)$ }

Fast Jones: {KnotTheory`TorusKnot [21, 2], 21, 0.656, 0.03125,  $-q^{10} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21})$ }

Fast Jones: {KnotTheory`TorusKnot [11, 3], 22, 0.719, 0.0625,  $q^{10} + q^{12} - q^{22}$ }

Fast Jones: {KnotTheory`TorusKnot [23, 2], 23, 0.766, 0.046875,  $-q^{11} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23})$ }

Slow Jones: {KnotTheory`TorusKnot [5, 3], 10, 0.812, 0.375,  $q^4 + q^6 - q^{10}$ }

Fast Jones: {KnotTheory`TorusKnot [6, 5], 24, 1.313, 0.546875,  $q^{10} (1 + q^2 + q^4 - q^7 - q^9)$ }

Fast Jones: {KnotTheory`TorusKnot [25, 2], 25, 1.391, 0.078125,  $-q^{12} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25})$ }

Fast Jones: {KnotTheory`TorusKnot [13, 3], 26, 1.485, 0.09375,  $q^{12} + q^{14} - q^{26}$ }

Fast Jones: {KnotTheory`TorusKnot [9, 4], 27, 1.719, 0.234375,  $-q^{12} (-1 - q^2 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 + q^{11})$ }

Fast Jones: {KnotTheory`TorusKnot [27, 2], 27, 1.797, 0.078125,  $-q^{13} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27})$ }

Slow Jones: {KnotTheory`TorusKnot [11, 2], 11, 2.25, 1.4375,  $-q^5 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11})$ }

Fast Jones: {KnotTheory`TorusKnot [7, 5], 28, 2.735, 0.9375,  $q^{12} (1 + q^2 + q^4 - q^8 - q^{10})$ }

Fast Jones: {KnotTheory`TorusKnot [14, 3], 28, 2.875, 0.125,  $q^{13} + q^{15} - q^{28}$ }

Fast Jones:

{KnotTheory`TorusKnot [29, 2], 29, 2.938, 0.0625,  $-q^{14} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29})$ }

Fast Jones:

{KnotTheory`TorusKnot [31, 2], 31, 3.031, 0.09375,  $-q^{15} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31})$ }

Fast Jones: {KnotTheory`TorusKnot [8, 5], 32, 4.313, 1.26563,  $q^{14} (1 + q^2 + q^4 - q^9 - q^{11})$ }

Fast Jones: {KnotTheory`TorusKnot [16, 3], 32, 4.453, 0.140625,  $q^{15} + q^{17} - q^{32}$ }

Fast Jones: {KnotTheory`TorusKnot [11, 4], 33,

4.938, 0.484375,  $-q^{15} (-1 - q^2 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} + q^{13})$ }

Fast Jones: {KnotTheory`TorusKnot [33, 2], 33, 5.047, 0.109375,

$-q^{16} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33})$ }

Fast Jones: {KnotTheory`TorusKnot [17, 3], 34, 5.219, 0.171875,  $q^{16} + q^{18} - q^{34}$ }

Fast Jones: {KnotTheory`TorusKnot [7, 6], 35, 9.75, 4.53125,  $-q^{15} (-1 - q^2 - q^4 - q^6 + q^7 + q^9 + q^{11})$ }

Fast Jones: {KnotTheory`TorusKnot [35, 2], 35, 9.86, 0.109375,

$-q^{17} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35})$ }

Fast Jones: {KnotTheory`TorusKnot [9, 5], 36, 11.391, 1.51563,  $q^{16} + q^{18} + q^{20} - q^{26} - q^{28}$ }

Fast Jones: {KnotTheory`TorusKnot [37, 2], 37, 11.5, 0.109375,

$-q^{18} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37})$ }

Slow Jones: {KnotTheory`TorusKnot [13, 2], 13, 11.547,

9.29688,  $-q^6 (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13})$ }

Fast Jones: {KnotTheory`TorusKnot [19, 3], 38, 11.703, 0.203125,  $q^{18} + q^{20} - q^{38}$ }

Fast Jones: {KnotTheory`TorusKnot [13, 4], 39, 12.297,

0.59375,  $-q^{18} (-1 - q^2 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} + q^{15})$ }

Fast Jones: {KnotTheory`TorusKnot [39, 2], 39, 12.422, 0.125,

$-q^{19} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39})$ }

Fast Jones: {KnotTheory`TorusKnot [20, 3], 40, 12.625, 0.203125,  $q^{19} + q^{21} - q^{40}$ }

Fast Jones: {KnotTheory`TorusKnot [41, 2], 41, 12.766, 0.125,

$-q^{20} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41})$ }

Fast Jones:

{KnotTheory`TorusKnot [43, 2], 43, 12.922, 0.15625,  $-q^{21} (-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43})$ }

Fast Jones: {KnotTheory`TorusKnot [11, 5], 44, 15.719, 2.79688,  $q^{20} + q^{22} + q^{24} - q^{32} - q^{34}$ }

Fast Jones: {KnotTheory`TorusKnot [22, 3], 44, 16.016, 0.296875,  $q^{21} + q^{23} - q^{44}$ }

Fast Jones: {KnotTheory`TorusKnot[15, 4], 45, 16.813, 0.796875,  
 $-q^{21}(-1 - q^2 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} + q^{17})$ }

Fast Jones:

{KnotTheory`TorusKnot[45, 2], 45, 17., 0.171875,  $-q^{22}(-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43} - q^{44} + q^{45})$ }

Fast Jones: {KnotTheory`TorusKnot[23, 3], 46, 17.313, 0.3125,  $q^{22} + q^{24} - q^{46}$ }

Fast Jones:

{KnotTheory`TorusKnot[47, 2], 47, 17.5, 0.1875,  $-q^{23}(-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43} - q^{44} + q^{45} - q^{46} + q^{47})$ }

Slow Jones: {KnotTheory`TorusKnot[7, 3], 14, 23.875, 12.3281,  $q^6 + q^8 - q^{14}$ }

Slow Jones: {KnotTheory`TorusKnot[5, 4], 15, 50.922, 27.0469,  $q^6(1 + q^2 + q^4 - q^5 - q^7)$ }

Fast Jones: {KnotTheory`TorusKnot[8, 7], 48, 51.61, 34.0938,  $q^{21}(1 + q^2 + q^4 + q^6 - q^9 - q^{11} - q^{13})$ }

Fast Jones: {KnotTheory`TorusKnot[12, 5], 48, 54.969, 3.34375,  $q^{22} + q^{24} + q^{26} - q^{35} - q^{37}$ }

Fast Jones:

{KnotTheory`TorusKnot[49, 2], 49, 55.156, 0.1875,  $-q^{24}(-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43} - q^{44} + q^{45} - q^{46} + q^{47} - q^{48} + q^{49})$ }

Fast Jones: {KnotTheory`TorusKnot[25, 3], 50, 55.531, 0.375,  $q^{24} + q^{26} - q^{50}$ }

Fast Jones: {KnotTheory`TorusKnot[17, 4], 51, 56.641, 1.10938,  
 $-q^{24}(-1 - q^2 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} + q^{19})$ }

Fast Jones: {KnotTheory`TorusKnot[51, 2], 51, 56.86, 0.203125,

$-q^{25}(-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43} - q^{44} + q^{45} - q^{46} + q^{47} - q^{48} + q^{49} - q^{50} + q^{51})$ }

Fast Jones: {KnotTheory`TorusKnot[13, 5], 52, 60.703, 3.84375,  $q^{24} + q^{26} + q^{28} - q^{38} - q^{40}$ }

Fast Jones: {KnotTheory`TorusKnot[26, 3], 52, 61.094, 0.390625,  $q^{25} + q^{27} - q^{52}$ }

Fast Jones: {KnotTheory`TorusKnot[53, 2], 53, 61.328, 0.21875,

$-q^{26}(-1 - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} - q^{12} + q^{13} - q^{14} + q^{15} - q^{16} + q^{17} - q^{18} + q^{19} - q^{20} + q^{21} - q^{22} + q^{23} - q^{24} + q^{25} - q^{26} + q^{27} - q^{28} + q^{29} - q^{30} + q^{31} - q^{32} + q^{33} - q^{34} + q^{35} - q^{36} + q^{37} - q^{38} + q^{39} - q^{40} + q^{41} - q^{42} + q^{43} - q^{44} + q^{45} - q^{46} + q^{47} - q^{48} + q^{49} - q^{50} + q^{51} - q^{52} + q^{53})$ }

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