1617-257 Wed Feb 8, hour 51: Alternating Tensors

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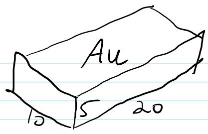

HW 13 due, HW 14 on web!
Read Along. Sections 27-29.
Riddle Along. The Cantor set C is of measure 0. Is it also true for C+C={x+y: x,y\in C}?
Is it always true that if B is of measure 0, then so is B+B?

$S_n := \{\text{bijections } \sigma : \underline{n} \circlearrowleft\}$ $\exists!\ \text{sign}: S_n \to \{\pm 1\}$ s.t. $(-1)^{\sigma\tau} = (-1)^\sigma (-1)^\tau$ $(-1)^{(ij)} = -1$.

Thm $\forall I \in \binom{\underline{n}}{k}$ $\exists!\ \psi_I \in A^k(V)$ s.t.

$\forall J \in \underline{n}^k_a$ $\psi_I(a_J) = \delta_{IJ}$; $\{\psi_I : I \in \binom{\underline{n}}{k}\}$ is a basis of $A^k(V)$ so $\dim A^k(V) = \binom{n}{k}$.
on board

Pf An alternating tensors is determined by its values on $\{a_J : J \in \binom{\underline{n}}{k}\}$ so if ψ_I exists, it is unique.

Existence: $\psi_I(x_1 \dots x_k) := \sum_{\tau \in S_n} (-1)^\tau \phi_I^\tau(x_1, \dots, x_k)$

also done — a dim 3 example

Remains to show: lin indep, span.

done line

Claim In $V = \mathbb{R}^n$ w/ $a_i = e_i$, $\psi_I(x_1 \dots x_k) = \det(X_I)$, w/ X_I = rows I of $X = (x_1 | \dots | x_k)$

In particular, if $I = (1 \dots n)$, $\psi_I(x_1 .. x_k) = \det(X)$.

So $= \det \begin{pmatrix} x_{1i_1} & x_{2i_1} & \cdots & x_{ki_1} \\ \vdots & & & \\ x_{1i_k} & \cdots & \cdots & x_{ki_k} \end{pmatrix}$

Proof. Both sides are multi-linear and alternating, so it is enough to verify equality on a_J.

Thm $\exists!$ op $\wedge : A^k(V) \times A^l(V) \to A^{k+l}(V)$ s.t.

1. $\wedge$ is associative & bilinear.
2. $\wedge$ is "super-symmetric".
3. $\psi_I = \phi_{i_1} \wedge \phi_{i_2} \wedge \dots \phi_{i_k}$

In addition, if $T : V \to W$, $T^*(f \wedge g) = T^*(f) \wedge T^*(g)$.

A tangent vector $\xi = (x, v)$ to $\mathbb{R}^n$; $T_x(\mathbb{R}^n)$ is a vector space.

Curve & tangents.

Tangents and directional derivatives: $D_\xi f$

Push forwards under $\alpha : \mathbb{R}^k \to \mathbb{R}^n$; covariance

$T_p(M)$ for a manifold M; curves, directional derivatives, pushforwards.

C^r vector fields.