1617-257 Thu Mar 30, hour 69: Stokes' theorem in low dimensions

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Read along: Sec 37-38. Eval rate: 31/92.

$\omega \in \Omega^{k-1}(M^k)$ (compact, oriented) $\Rightarrow \int_M d\omega = \int_{\partial M} \omega$

"de-Rham complex": $\Omega^0(\mathbb{R}^3) \xrightarrow{d} \Omega^1(\mathbb{R}^3) \xrightarrow{d} \Omega^2(\mathbb{R}^3) \longrightarrow \Omega^3(\mathbb{R}^3)$

$\{f\}$ $\quad \sum a_i dx_i \quad b_1 dx_2 \wedge dx_3$ + cyc perms $\quad c\, dx_3$

$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$

vector calculus: $\{f\} \xrightarrow{\text{grad}} \left\{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right\} \xrightarrow{\text{curl}} \left\{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right\} \xrightarrow{\text{div}} \{c\}$

manifolds: points w/ signs $\cup (s_i, p_i)$ $\quad$ curves $\Gamma = \text{im}(\gamma: [0,1] \to \mathbb{R}^3)$ $\quad$ surfaces $S = \text{im}(\sigma: D^2 \to \mathbb{R}^3)$ $\quad$ solids/domains $D \subset \mathbb{R}^3$

on board

integration: $\int_M \omega \qquad \sum s_i f(p_i) \qquad \int_0^1 \vec{a}\cdot\dot{\gamma} = \int_\Gamma \vec{a}\cdot\vec{T}\,dV \qquad \int_S b\cdot\vec{n}\,dV \qquad \int_D c$

Stokes:

$$\int_0^1 (\text{grad} f)\cdot\dot{\gamma} = f(\gamma(1)) - f(\gamma(0))$$

Stoke's:

$$\int_S (\text{curl}\, a)\cdot\vec{n}\,dV = \int_{\partial S} \vec{a}\cdot\vec{T}\,dV$$

Stokes:

$$\int_D \text{div}\, b = \int_{\partial D} b\cdot\vec{n}\,dV$$

Aside

$$\sigma^*(b_1 dx_{23} + \text{cyc perms}) = \Big(b_1 dx_{23}\big(\underset{\begin{smallmatrix}\partial_x\sigma_1\\ \partial_x\sigma_2\\ \partial_x\sigma_3\end{smallmatrix}}{\sigma_* e_1}, \underset{\begin{smallmatrix}\partial_y\sigma_1\\ \partial_y\sigma_2\\ \partial_y\sigma_3\end{smallmatrix}}{\sigma_* e_2}\big) + cp\Big) dx\wedge dy$$

$$= \big(b_1(\partial_x\sigma_2 - \partial_y\sigma_3) + cp\big)\,dx\wedge dy = b\cdot[(\partial_x\sigma)\times(\partial_y\sigma)]\,dx\wedge dy$$

$$= b\cdot\vec{n}\cdot\text{Vol}(\partial_x\sigma, \partial_y\sigma)\,dx\wedge dy$$

done line

What is Stokes' Thm in each of these circumstances?

It is what it is: $\int_M d\omega = \int_{\partial M} \omega$

Really we're answering "translate beauty to old-fashioned ugliness".