1617-257 Mon Mar 6, hour 59: d and 3D

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Read Along: Sections 29-31. TT3: Much like TT1 and TT2, details on Wed.

$$\begin{array}{ccccccc} \Omega^0(\mathbb{R}^3) & \xrightarrow{d} & \Omega^1(\mathbb{R}^3) & & \Omega^2(\mathbb{R}^3) & \xrightarrow{d} & \Omega^3(\mathbb{R}^3) \\ \{f\} & & \{a_1dx_1+a_2dx_2+a_3dx_3\} & & \{b_1dx^2\wedge dx^3 + \text{cyc perms}\} & & \{c\,dx_1\wedge dx_2\wedge dx^3\} \\ \updownarrow & & \updownarrow & & & & \\ \{f\} & \xrightarrow[\nabla]{\text{grad}} & \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} & & \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} & \xrightarrow[\nabla\cdot]{\text{div}} & \{c\} \end{array}$$

Theorem. $\exists!$ linear operator $d: \Omega^k(\mathbb{R}^n) \longrightarrow \Omega^{k+1}(\mathbb{R}^n)$ s.t.

1. If f is a 0-form, df is as above.
2. $\omega\in\Omega^k$, $\eta\in\Omega^l \Rightarrow d(\omega\wedge\eta) = (d\omega)\wedge\eta + (-1)^k\omega\wedge d\eta$
3. $d^2=0$; more precisely, $d(d\omega)=0$.

on board

Pf 1-3 imply

$$d(\underset{\omega}{\underset{\parallel}{\sum a_I\, dx_I}}) = \sum_{j=1}^n \sum_I \frac{\partial a_I}{\partial x_j} dx_j\wedge dx_I \overset{\text{loosely}}{=} \sum_{j=1}^n dx_j\wedge\frac{\partial \omega}{\partial x_j}$$

* Continue w/ 3D diagram.
* Geom. interp. of grad, div, curl.
* Finish proof of Thm.