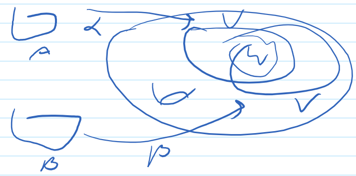
1617-257 Mon Mar 20, hour 65: Orientation, Integration

February 15, 2017 12:58 PM

TT Return at 2:50. Read along: Sec 33-35 and 37.



$\oint^{\alpha}_{M}\omega = \pm \oint^{\beta}_{M}\omega$

not task.

Definition An orientation on M is a cont./consistent choice of a pos-det-class of basis on T_xM, for every $x\in M$. "orientable" "oriented"

Claim If M is connected, it has 0 or 2 orientations.

Claim If $M^k\subset\mathbb{R}^{k+1}$, orientation $\Longleftrightarrow$ "sides"

Claim If M is oriented, then so is ∂M by "∂M's orientation is such that if you prepend to it the positive outgoing normal to ∂M, you get the orientation of M."

on board.

Example 1. orient ∂D^3 at $\begin{pmatrix}1\\0\\0\end{pmatrix}$. Example 2. orient ∂A, $A=\{x\in\mathbb{R}^2 : 1\le\|x\|\le 2\}$, assuming A inherits the std orientation of $\mathbb{R}^2$.

Orientation preserving maps, "positive charts", composition of such.

If M is oriented & α & β are positive, $\oint^{\alpha}_{M}\omega = \oint^{\beta}_{M}\omega =: \int_M\omega$

done

Loose def of $\int_M\omega$ 1. Using PO1

2. By chopping in pieces w/ meas-0 exceptions.

Example Let S^2 be oriented as ∂D^3 and let $\omega\in\Omega^2(\mathbb{R}^3)$ be

$\omega = x\,dy\wedge dz + y\,dz\wedge dx + z\,dx\wedge dy$. Compute $\int_{S^2}\omega = \int_{S^2} i^*\omega$.

$\alpha : [0,\infty)_r\times[0,2\pi]_\theta\times[-\frac{\pi}{2},\frac{\pi}{2}]_\phi$ be $\alpha(r,\theta,\phi) = (r\cos\theta\cos\phi, r\sin\theta\cos\phi, r\sin\phi)$

$\det D\alpha = r^2\cos\phi > 0$ so $\beta : [0,2\pi]_\theta\times[-\frac{\pi}{2},\frac{\pi}{2}]_\phi \to S^2$ by

$\beta(\theta,\phi) = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$ is positive.

$\beta^* i^*\omega = \cos\theta\cos\phi\,(d\sin\theta\cos\phi)\wedge(d\sin\phi) + \sin\theta\cos\phi\,(d\sin\phi)\wedge d(\cos\theta\cos\phi) + \sin\phi\,(d\cos\theta\cos\phi)\wedge(d\sin\theta\cos\phi)$

$= d\theta\wedge d\phi\,(\cos^2\theta\cos^3\phi + \sin^2\theta\cos^3\phi + \sin^2\phi\sin^2\theta\cos\phi + \sin^2\phi\cos^2\theta\cos\phi)$

$= d\theta\wedge d\phi\,(\cos^3\phi + \cos\phi\sin^2\phi) = \cos\phi\, d\theta\wedge d\phi$

So $\int_M\omega = \int_{[0,2\pi]_\theta\times[-\frac{\pi}{2},\frac{\pi}{2}]_\phi}\cos\phi\, d\theta d\phi = 4\pi$