1617-257 Mon Mar 13, hour 62: pullbacks, determinants, integration

February 15, 2017 12:58 PM

Read along: Sec 32-34.
TT: Tue March 14 5PM-7PM @ EX 300. Extra OH: Dror Mon March 13 5-8PM BA 6178, Jeff Tue March 14 11-2 Huron 215 10th floor.

Claim If $\phi:\mathbb{R}^n_{x_i}\to\mathbb{R}^n_{y_j}$ & $\omega=f\,dy_{\underline{n}}\in\Omega^{top}(\mathbb{R}^n)$, then $\phi^*(\omega)=\overbrace{\det(D\phi)}^{J_\phi}\cdot\phi^*f\cdot dx_{\underline{n}}$

Example $\mathbb{R}^2_{r\theta}\xrightarrow[\binom{r\cos\theta}{r\sin\theta}]{\phi}\mathbb{R}^2_{x,y}$ as above; $\omega=dx\wedge dy$; $\phi^*\omega=J_\phi\,dr\wedge d\theta=r\,dr\wedge d\theta$

on board

proof use $\psi_I(x_1,\dots,x_k)=\det(X_I)$, w/ X_I = rows I of $X=(x_1|\dots|x_k)$ from a while ago:

$\Rightarrow dx_{\underline{n}}(v_1\dots v_n)=\det(v_1|\dots|v_n)$ so

$\phi^*(\omega)(e_1\dots e_n)=\omega(\phi_*e_1\dots\phi_*e_n)=f(x)\cdot\det(D\phi)$. This is also the r.h.s.

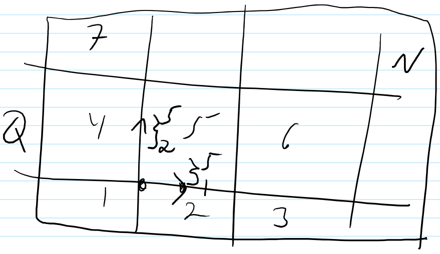
Claim If $\phi:\mathbb{R}^n_{x_i}\to\mathbb{R}^m_{y_j}$ & $\omega=\sum a_I\,dy_I\in\Omega^k(\mathbb{R}^m)$, then

$$\phi^*(\omega)=\sum_{I\in\binom{m}{k}}\sum_{J\in\binom{n}{k}}\phi^*(a_I)\cdot\det\left(D\phi(x)_{J,I}\right)\cdot dx_J$$

The J rows & I col's of $D\phi(x)$.

$\omega\in\Omega^{top}(\mathbb{R}^k)$ is always $f\,dx^1\wedge\dots\wedge dx^k$. Define $\int_{\mathbb{R}^k}\omega=\int_{\mathbb{R}^k}f$ (assuming supp f is small)

* This makes sense!



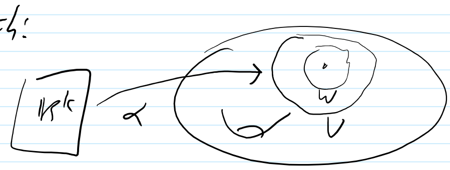
$\int_Q\omega\sim\sum_{i=1}^N\omega(\xi_1^i,\xi_2^i)=\dots$

"Intuitive"

Let $\omega\in\Omega^k(M)$ be a k-form on a k-manifold in $\mathbb{R}^n$ whose support lies within the image of one coord. patch:

Define $\int^{\alpha}_M\omega=\int_{\mathbb{R}^k}\alpha^*\omega$

preliminary



done line

Claim If $\beta:\mathbb{R}^k\to V\subset\mathbb{R}^n$ is a patch also containing supp ω, then

$$\int^{\alpha}_M\omega=\pm\int^{\beta}_M\omega$$

(Note: $U\cap V$ should be connected)

stated, not proven

our next battle, "orientations".