1617-257 Mon Feb 13, hour 53: Wedge products

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Read Along. Sections 27-29.

Riddle Along. Ahmed and Betty live in different towns and have good 19th century hardware (paper, pens, envelopes, boxes, locks, keys, but nothing electronic). They'd like to communicate privately via an untrusted intermediate, Dror, who routinely travels between their towns. Can they do that, even if they haven't coordinated a code or exchanged keys in advance?

$f\in A^k(V)$, $g\in A^l(V)$, define $f\wedge g\in A^{k+l}(V)$ via

$$(f\wedge g)(x_1\dots x_{k+l}) := \frac{1}{k!\,l!}\sum_{\sigma\in S_{k+l}} (-1)^\sigma f(x_{\sigma 1}\dots x_{\sigma k})\,g(x_{\sigma(k+1)},\dots x_{\sigma(k+l)})$$

$$= \sum_{\substack{\sigma\in S_{k+l}\\ \sigma 1<\dots<\sigma k\\ \sigma(k+1)<\dots<\sigma(k+l)}} (-1)^\sigma f(x_{\sigma 1}\dots x_{\sigma k})\,g(x_{\sigma(k+1)},\dots x_{\sigma(k+l)})$$

on board

Thm $\exists!$ op $\wedge: A^k(V)\times A^l(V)\longrightarrow A^{k+l}(V)$ s.t.

1. $\wedge$ is associative & bilinear.
2. $\wedge$ is "super-symmetric".
3. $\psi_I = \phi_{i_1}\wedge\phi_{i_2}\wedge\dots\phi_{i_k}$

Also, if $T: V\longrightarrow W$, then $T^*: A^k(W)\to A^k(V)$ and $T^*(f\wedge g) = T^*(f)\wedge T^*(g)$.

A tangent vector $\xi = (x, v)$ to $\mathbb{R}^n$; $T_x(\mathbb{R}^n)$ is a vector space.

Curve & tangents.

Tangents and directional derivatives: $D_\xi f$

Push forwards under $\alpha: \mathbb{R}^k\to\mathbb{R}^n$; covariance

$T_p(M)$ for a manifold M; curves, directional derivatives, pushforwards.

C^r vector fields.

add details?