1617-257 Fri Mar 31, hour 70: Gauss, Closed, exact, Poincare, Hodge

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Read along: don't. Eval rate: 36/92.

b: v.f. c: function S: Surface D: Domain/solid.

$\omega_2(b) := b_1\, dx_{23} + c.p.$ $\int_S \omega_2(b) = \int_S b\cdot\vec{n}\,dV$

$\omega_3(c) := c\, dx_{\underline{3}}$ $\int_D \omega_3(c) = \int_D c$

$d\omega_2(b) = \omega_3(div(c))$. Stokes' w/ $M \to D$, $\omega \to \omega_2(b)$ becomes.

start here

$$\int_D div\, b = \int_D \omega_3(div\, b) = \int_D d\omega_2(b) = \int_D d\omega = \int_{\partial D} \omega = \int_{\partial D} \omega_2(b) = \int_{\partial D} b\cdot\vec{n}\,d\vec{v}$$

"fluid creation within D" "flow out through ∂D"

<u>Def</u> $\omega \in \Omega^*(M)$ is "closed" if $d\omega = 0$

is "exact" if $\exists \lambda$ s.t. $d\lambda = \omega$

<u>Claim</u> Every exact form is closed

<u>Example</u> $\omega = \dfrac{x\,dy - y\,dx}{x^2+y^2}$ is closed but not exact.

* closed by computation.
* closed by pullback to r, θ
* closed as $d(\arctan y/x)$
* not exact.

Poincaré's lemma: on $\mathbb{R}^n$, closed $\Rightarrow$ exact.

"de-Rham" on any manifold, closed is "almost exact"; namely,

$$H^k_{dR}(M) := \{\text{closed }k\text{-forms}\} \big/ \{\text{exact }k\text{ forms}\}$$ is finite dimensional.

... the beginning of a long story...

End of Final-exam class material

~~An inner product on $\Lambda^k(V)$ from an inner product on V~~ done fine (in practice $\langle dx_I, dx_J \rangle = \delta_{IJ}$)

Hodge: For every $\omega \in \Omega^k$, let $*\omega$ be the $(n-k)$ form s.t.

$$\langle \omega, \eta \rangle dx_{\mathbb{R}} = (*\omega) \wedge \eta$$