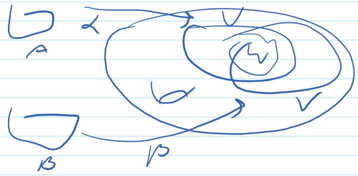
1617-257 Fri Mar 17, hour 64: Orientations

February 15, 2017 12:58 PM

Read along: Sec 33-35.
No class Monday March 27, makeup Thursday March 30 5PM @ MP134, also on video.



$\overset{\alpha}{\oint_M} \omega = \pm \overset{\beta}{\oint_M} \omega$

next task.

Definition An orientation $\mathcal{O}$ on a f.d. v.s. V is a choice of an ordered basis for V, regarded up to positive-det changes of bases:

$$(v_1 \dots v_n) \sim (u_1 \dots u_n) \quad \text{if } \det(C_v^u) > 0.$$

Every v.s. has precisely two orientations. on board

Def An orientation on a manifold M is a cont. choice of orientations $\mathcal{O}_x$ on T_xM, for every $x \in M$.

Cont: Every $p \in M$ has a nbd W with cont. vector fields $u_1 \dots u_k$ on W s.t. for every $x \in U$, $(u_1(x), \dots u_k(x)) \sim \mathcal{O}_x$.

Examples:

"non-orientable"

1. If $M^k \subset \mathbb{R}^{k+1}$ and $\mathbb{R}^{k+1}$ is oriented, then there is a bijection

$$\left\{ \begin{matrix} \text{orientation of} \\ M \end{matrix} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{Cont. varying choice of unit} \\ \text{normal vectors to } M \end{matrix} \right\}$$

$$\left(:= \begin{matrix} \forall p \in M \text{ a vector } n(p) \in T_p\mathbb{R}^{k+1} \text{ s.t.} \\ 1.\ n(p) \perp T_pM \\ 2.\ p \mapsto n(p) \text{ is cont.} \end{matrix} \right)$$

Pf...

2. If M is oriented, so is ∂M. } repeat.

done line

3. Orientation preserving maps, "positive charts"
4. If M is oriented & α & β are positive, $\overset{\alpha}{\oint_M} \omega = \overset{\beta}{\oint_M} \omega$