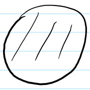
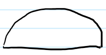
1617-257 Fri Jan 27, hour 46 draft

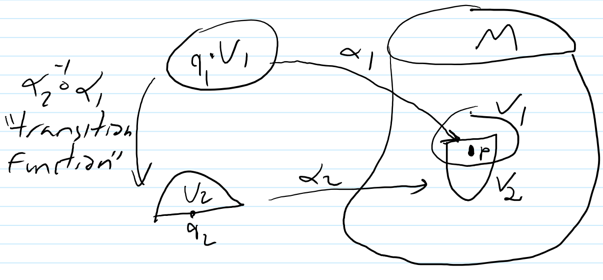
January 16, 2017 8:21 AM

$$\int_M d\omega = \int_{\partial M} \omega$$

- Read Along: 24-25
- Riddle Along: The game of 15 is played as follows. Two players alternate choosing cards numbered between 1 and 9, with repetitions forbidden, so the game ends at most after 9 moves (or 4 and a half rounds). The first player to have within their cards a set of precisely 3 cards that add up to 15 wins. Would you rather move first or second?

To do: Last class in pictures

Q Is  diffeo to  Aside Is  diffeo to  ?



$V_1 \cap V_2$ in an open nbd of p in M

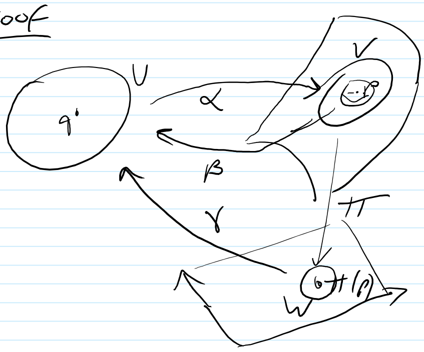
Let $U_1' = \alpha_1^{-1}(V_1\cap V_2)$ $U_2' = \alpha_2^{-1}(V_1 \cap V_2)$

these are open nbds of q_1, q_2 in H^k

$\alpha_2^{-1}\circ\alpha_1 : U_1' \to U_2'$ is a homeo mapping an inner point to an edge point; this is impossible but very hard to show. If only we knew α_1^{-1} & α_2^{-1} were diffable.... We'll only do α_1^{-1}; α_2^{-1} is similar (and is in book)

<u>Proposition</u> If M^k is a class C^r manifold in $\mathbb{R}^n$ and $\alpha: U \subset \mathbb{R}^k \to V \subset M^k$ is a coordinate patch, then $\alpha^{-1}: V \to U$ is C^r.

<u>Proof</u>



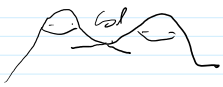
NTS: there is a C^r function $\beta: V' \to U'$, s.t. V' is a nbd of p in $\mathbb{R}^n$, U' is a nbd of q contained in U, and $\beta\circ\alpha = Id$ on U'.

<u>Pf</u>: For convenience, assume the first k rows of $d\alpha(q)$ are lin. indep., and let $\pi: \mathbb{R}^n \to \mathbb{R}^k$ be the proj. on the first k coords. Then $d(\pi\circ\alpha)(q)$ is invertible, so $\pi\circ\alpha$ has an inverse γ near $\pi(p)$; precisely, on some nbd W of $\pi(p)$ for which $(\pi\circ\alpha)^{-1}(W) \subset U$.

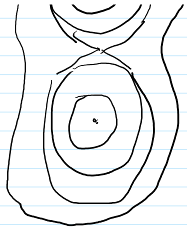
Now let $V' = \pi^{-1}(W)$, $U' = (\pi\circ\alpha)^{-1}(W) = \alpha^{-1}(M\cap V')$ and $\beta = \gamma\circ\pi$.

<u>Corollary</u> "transition functions" are C^r.

Theorems we skip:

1. ∂M is a $(k-1)$-manifold.
2.  

If $A \subset \mathbb{R}^n$ is open and $f: A \to \mathbb{R}$ is C^r, ~~then for most h $f^{-1}(h)$ is a manifold.~~ and $h \in \mathbb{R}$ is such that whenever $p \in f^{-1}(h)$, $df(p)$ has



and $h \in \mathbb{R}$ is such that whenever $p \in f^{-1}(h)$, $df(p)$ has rank 1, then $f^{-1}(h) = N$ is a manifold,
and so are $f^{-1}((-\infty, h]) = M_1$ & $f^{-1}([h, \infty)) = M_2$,
and $\partial M_1 = \partial M_2 = N$

Corollary: S^{n-1} is a mfld & $S^{n-1} = \partial D^n$.