Dror Bar-Natan: Academic Pensieve: Classes: 1617-257b-AnalysisII:

1617-257 Fri Feb 17, hour 55: Tangent Vectors

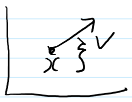
February 15, 2017 12:58 PM

Our final: Thu Apr 20 2-5PM at HA 403 (Haultain Building, the most unfindable on campus).
Read Along: sections 29-30.

I was asked to read to you the paragraph below. If you are interested, contact me after class.

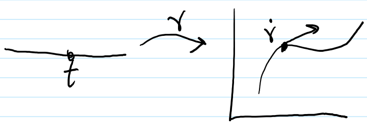
The Association of Part-Time Undergraduate Students or APUS represents all part-time undergraduate students at the University. They offer students cost-saving services, advocacy and events. APUS is currently seeking part-time students to volunteer as class representatives. As a Class Representative for this course, you can help shape the work of the students' union. You will also keep other part-time students aware of the services, events, and campaigns to which students have access and can get involved, such as the health and dental plan, bursaries, campaigns on tuition fees, and events throughout the year. If anyone is interested in being the class representative for this course, please indicate your interest to get involved with the part-time students' union.

Riddle Along. $\mathcal{C}\left(\underset{-1}{\times} \text{—} z_1 \text{—} z_2 \text{—} \underset{1}{\times}\right) = \left\{ (z_1, z_2) \in \mathbb{C}^2 : \begin{matrix} d(-1, z_1) = d(z_1, z_2) \\ = d(z_2, 1) = 1 \end{matrix} \right\} \simeq ?$

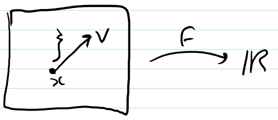


Tangent vector: $\xi = (x, v)$

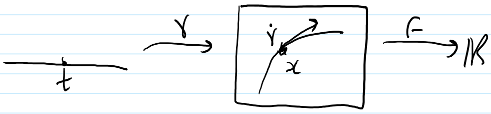
$T_x(\mathbb{R}^n) = \{\xi = (x, v)\}$, a vector space w/ ops acting only on the v-part.



$\dot\gamma(t) = (\gamma(t), D\gamma_t(e_1))$



$D_\xi f := Df_x(v)$



$\Rightarrow \frac{d(f\circ\gamma)}{dt} = D_{\dot\gamma(t)} f$

on board.

Push forwards under $\alpha: \mathbb{R}^k \to \mathbb{R}^n$; covariance

$T_p(M)$ for a manifold M; curves, directional derivatives, pushforwards.

C^r vector fields.

$Y: A \subset \mathbb{R}^n \longrightarrow \ldots \quad Y(x) \in T_x\mathbb{R}^n \sim \mathbb{R}^n$, so $Y(x) = \sum y_i(x) e_i$

f function $\longrightarrow Yf$ by $Yf(x) = D_{Y(x)} f = \sum y_i(x) \frac{\partial f}{\partial x_i}$

Y is C^r means $\forall i\ y_i \in C^r \iff [f \in C^{r+1} \Rightarrow Yf \in C^r]$

done line, sans proof of last point.

A vector field on a manifold $M \subset \mathbb{R}^n$: $Y = \sum y_i(x) e_i$, s.t. $\forall x\ Y(x) \in T_x M$

Do not push or pull!

k-Tensor fields on open $A \subset \mathbb{R}^n$ $\quad W: A \longrightarrow \text{mess}\ \left[= \bigcup_x \mathcal{L}^k(T_x\mathbb{R}^n)\right]$

s.t. $W(x) \in \mathcal{L}^k(T_x\mathbb{R}^n)$.

So if $\xi_i \in T_x\mathbb{R}^n$, $W(\xi_1 \ldots \xi_k) = W(x)(v_1 \ldots v_k)$ makes sense.

So if $Y_1 \ldots Y_k$ are v.f., $W(Y_1 \ldots Y_k)$ is a function.

k-form on an open $A \subset \mathbb{R}^n$

same, but w/ target space $A^k(T_x\mathbb{R}^n)$

In practice, identify all $T_x\mathbb{R}^n$ w/ $\mathbb{R}^n$, and then

$$W = \sum_{I \in n^k} a_I(x)\, \phi_I(x) \quad \text{or} \quad W = \sum_{I \in \binom{n}{k}} a_I\, \psi_I(x)$$

W is C^r means $\forall I\ a_I \in C^r \Leftrightarrow \forall C^r\ Y_1 \dots Y_k,\ W(Y_1 \dots Y_k) \in C^r$