1617-257 Fri Feb 10, hour 52: Determinants, wedge products

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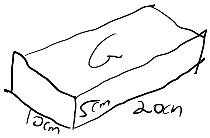

Read Along. Sections 27-29.
Riddle Along. Can you hang a medal on two nails so that if you remove any one of them, the medal will fall? Can you do the same with 3 nails?

Thm $\forall I \in \binom{n}{k}\ \exists!\ \psi_I \in A^k(V)$ s.t.

$\forall J \in \binom{n}{k}$ $\psi_I(a_J)=\delta_{IJ}$; $\{\psi_I : I \in \binom{n}{k}\}$ is a basis of $A^k(V)$ so $\dim A^k(V) = \binom{n}{k}$.

on board

Claim In $V=\mathbb{R}^n$ w/ $a_i=e_i$, $\psi_I(x_1 \dots x_k) = \det(X_I)$, w/ X_I = rows I of $X=(x_1|\dots|x_k)$

In particular, if $I=(1\dots n)$, $\psi_I(x_1 \dots x_k)=\det(X)$. So $= \det\begin{pmatrix} x_{1i_1} & x_{2i_1} & \dots & x_{ki_1} \\ \vdots & & & \\ x_{1i_k} & - & - & x_{ki_k} \end{pmatrix}$

Proof. Both sides are multi-linear and alternating, so it is enough to verify equality on a_J.

Thm $\exists!$ op $\wedge : A^k(V) \times A^l(V) \to A^{k+l}(V)$ s.t.

±/done

($\wedge$ defined by formula, not by thm)

1. $\wedge$ is associative & bilinear.
2. $\wedge$ is "super-symmetric".
3. $\psi_I = \phi_{i_1} \wedge \phi_{i_2} \wedge \dots \phi_{i_k}$

In addition, if $T: V \to W$, $T^*(f \wedge g) = T^*(f) \wedge T^*(g)$.

A tangent vector $\xi=(x,v)$ to $\mathbb{R}^n$; $T_x(\mathbb{R}^n)$ is a vector space.

Curve & tangents.

tangents and directional derivatives: $D_\xi f$

Push forwards under $\alpha: \mathbb{R}^k \to \mathbb{R}^n$; covariance

$T_p(M)$ for a manifold M; curves, directional derivatives, pushforwards.

C^r vector fields.