TT2: Q3 and Q6 Marking Key

Problem 3.

1. Very carefully, define "a bounded function f is integrable on a rectangle Q". Assume that your reader does not know the words / phrases "partition of an interval", "partition of Q", "a rectangle in a partition", "the volume of a rectangle", "lower sum", "upper sum", and whatever remains.

2. Directly from the definitions, prove that constant functions are integrable.

Marking Key. 18 for part 1, of which 9 global and 9 local. 7 for part 2.
(5/25) Fully correct quote/use of "f integrable iff $\mu(D(f)) = 0$".
(−3) The statement $\sum V(R_i) = V(Q)$ is skipped.
(−4) Reverse-order definitions.
(−2) Min/max instead of inf/sup.
(−1) Restricted to 2D.

Problem 6. Let $x_1, \ldots, x_k \in \mathbb{R}^n$. Prove:

1. Interchanging two of these vectors does not change the volume of the parallelepiped they span.

2. Adding a multiple of one of those vectors to another one does not change the volume of the parallelepiped they span.

3. Multiplying one of these vectors by a scalar $\lambda \in \mathbb{R}$ multiplies the volume of the parallelepiped they span by $|\lambda|$.

4. If these vectors are orthonormal, the volume of the parallelepiped they span is 1.

Tip. For this problem, you may freely use everything stated in class.

Marking Key. 5 points for using a reasonable volume formula or definition. Then 5 points for each part.
6/25 if pretended that $k = n$.
(−15) All hinges on $\det(AB) = \det(A)\det(B)$ for non-square matrices.
(−2) The action on $X^T X$ is just on rows/columns, rather than both.
(−3) Column actions by $X \mapsto EX$, then cancelled middle terms in $\det(X^T E^T EX)$.