Homework Assignment 11

Reading. Read, reread and rereread your notes to this point, and make sure that you really, really really, really really really understand everything in them. Do the same every week! Also, read, reread and rereread sections 17 and 20-24 of Munkres' book to the same standard of understanding. Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read. Also, preread section 25, just to get a feel for the future.

Doing. Solve problems 6 and 7 in section 17, problem 2 in section 20, problems 3 and 5 in section 21, and problems 1 and 2 in section 22, but submit only your solutions of the underlined problems. In addition, solve the following two exercises, but submit only your solution of part 1 of exercise 2.

Hint for Problem 2 in Section 22. In some approaches, it may be possible to simplify the end result using the formula $\det(I_{n\times n} + vw^T) = 1 + v^T w$, which holds for vectors $v, w \in \mathbb{R}^n$.

Exercise 1: Curves. Consider a simple curve given by a smooth map $\gamma : [0,T] \to \mathbb{R}^n$, and let $C := \operatorname{Im}(\gamma)$.

1) Prove that the arclength $\lambda(C)$ of the curve C is given by $\lambda(C) = \int_0^T \|\dot\gamma(t)\| dt$, where $\|\cdot\|$ is the Euclidian norm on $\mathbb{R}^n$ and $\dot\gamma = \frac{d}{dt}\gamma$.

2) Consider the length at time $t \in [0,T]$ of γ, $s(t) = \int_0^t \|\dot\gamma(u)\| du$. Let $\hat\gamma : [0,\lambda(C)] \to \mathbb{R}^n$ be the reparametrization of C in terms of arclength (i.e. $C = \operatorname{Im}(\hat\gamma)$). Supposing that $f : \mathbb{R}^n \to \mathbb{R}$ is a function that is continuous in a neighborhood of C, express the integral $\int_C f$ in terms of the parametrizations γ and $\hat\gamma$.

3) Consider the helix $C \subset \mathbb{R}^3$ parametrized as follows:

$$\gamma(t) = \begin{pmatrix} r\cos t \\ r\sin t \\ 2t \end{pmatrix}, \; t \in [-2\pi, 2\pi].$$

Compute the arclength $\lambda(C)$ and the integral $\int_C f$ where $f(x,y,z) = xy\sin z$.

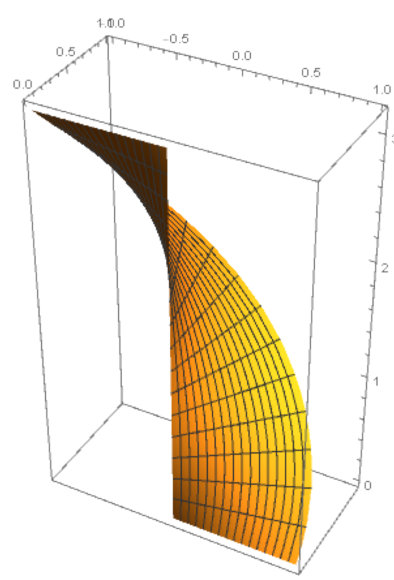
Exercise 2: Surfaces. A parametrization of an orientable surface $\mathcal{S} \subset \mathbb{R}^3$ is a C^r map $\sigma : \mathcal{U} \longrightarrow \mathbb{R}^3, (u,v) \longmapsto (x(u,v), y(u,v), z(u,v))$, where $\mathcal{U} \subset \mathbb{R}^2$ is an open subset.

1) Compute the area $\mathcal{A}(\mathcal{S})$ of the surface $\mathcal{S} \subset \mathbb{R}^3$ given by the parametrization:

$$\sigma(u,v) = (u\cos v, u\sin v, v), \; \forall (u,v) \in (0,1)\times(0,\pi).$$

2) Let $\mathcal{S}$ be the part of the sphere $x^2+y^2+z^2 = a^2$ lying inside the cylinder $x^2+y^2 = b^2$ where $0 < b < a$. Compute the surface area $\mathcal{A}(\mathcal{S})$.

```
ParametricPlot3D[{u Cos[v], u Sin[v], v},
 {u, 0, 1}, {v, 0, π}]
```



Submission. Here and everywhere, **neatness counts!!** You may be brilliant and you may mean just the right things, but if the teaching assistants will be having hard time deciphering your work they will give up and assume it is wrong. This assignment is due in class on Wednesday January 25 by 2:10PM. Please write on your assignment the day of the tutorial when you'd like to pick it up once it is marked (Wednesday or Thursday).