1617-257 Wed Oct 26, Hour 19: Inverse Functions, 4; implicit functions

October 14, 2016 6:25 AM

Riddle Along: Can you pack 21 3x1 rectangles and on an 8x8 board? Are there limits on where the missing piece may be?
Read Along: Secs 8,9.
TT: Tue Nov 1 5PM-7PM @ BI 131. Extra OH: Jeff Mon 4-7 Huron 215 10th floor, Dror Tue 11-2 BA 6178,
Agenda: The Inverse Function Theorem.

Thm (IFT) $f:\mathbb{R}^n \circlearrowleft$ is C^1 near $a\in\mathbb{R}^n$, $\exists\, Df(a)^{-1}$
$\Rightarrow \exists$ nbds $U\ni a$, $V\ni b=f(a)$ s.t. $\exists (f|_U)^{-1}: V\to U$;
$f\in C^r \Rightarrow f|_U^{-1}\in C^r$. WLOG $Df(a)=0$, $a=b=0$.

TL f is Jolly-rigid near a: $\forall\epsilon>0\ \exists$ nbd $J_\epsilon\ni a$ s.t.
$$\forall x,y\in J_\epsilon\quad \|\underbrace{f(y)-f(x)}_{u}-\underbrace{(y-x)}_{v}\|\le \epsilon\|y-x\| \le \cancel{\tfrac{\epsilon}{1-\epsilon}}\,\epsilon\,\|f(y)-f(x)\|$$

Done: $V=0.4J_{0.1}$, $U=f^{-1}(V)$, $(f|_U)^{-1}$ exists & cont.

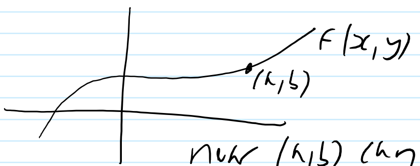
on board

TT Details:

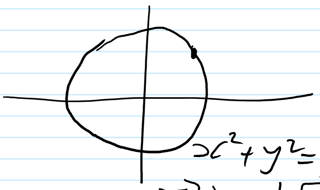
- Material: Everything to Friday, roughly proportional to time spent.
- Roughly choose 4/5, some questions multi-part.
- About 1/3 "prove as in class", 1/3 "solve as in HW", 1/3 "solve fresh".
- How I used to prepare.

Part IV f^{-1} is diffable at 0.
Part V f^{-1} is diffable near 0.
Part VI f^{-1} is C^r.

The implicit function thm



near (a,b) can write $y=g(x)$ s.t. $f(x,g(x))=0$.



$x^2+y^2=1$
$\Rightarrow y=\pm\sqrt{1-x^2}$
Sometimes two sol's
Sometimes none

Thm Given a C^r $f:\mathbb{R}^n_{x_1\ldots x_n}\times\mathbb{R}^k_{y_1\ldots y_k}\longrightarrow\mathbb{R}^k$ and $(a,b)\in\mathbb{R}^n\times\mathbb{R}^k$ s.t. $f(a,b)=0$
&
there exists a unique C^r $g:\{\text{nbd } U \text{ of } a\}\to\{\text{nbd of } b\}$ s.t. $g(a)=b$ & $\forall z\in U$ $f(z,g(z))=0$.
Furthermore, $Dg=$

done line

pf $f(z,y)=0 \iff \begin{cases} x=z\\ f(x,y)=0\end{cases}$ so with $H(x,y):=\begin{pmatrix}x\\ y\end{pmatrix}$

this is $H\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}z\\0\end{pmatrix}$ where $H\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}a\\0\end{pmatrix}$. Assuming $DH\begin{pmatrix}a\\b\end{pmatrix}$ is non-singular,
H^{-1} exists near $\begin{pmatrix}a\\0\end{pmatrix}$. So for z near a, $\exists\begin{pmatrix}x\\y\end{pmatrix}$ s.t. $H\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}z\\0\end{pmatrix}$...u
so set $g(z):=\pi_2\circ H^{-1}\begin{pmatrix}z\\0\end{pmatrix}$

Q: When is $DH(a)$ invertible?

so set $g(z) := \pi_2 \circ H \binom{z}{0}$

* When is $DH\binom{a}{b}$ invertible?
* What is Dg?