1617-257 Mon Oct 3, Hour 10: Compactness, connectedness

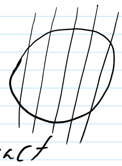
September 18, 2016 3:11 PM

Class photo on web; add your name!

Read Along: sect 4-5

Today's agenda: compactness, connectedness.

Riddle Along: A spherical loaf of bread is put into a bread slicing machine. Which slice gets the most crust?



The extremal value theorem: A cont. fnctn on a compact set attains its inf and its sup.

<u>Def</u> X is "connected" if the only clopen sets in X are $\emptyset$ & X $\Leftrightarrow$ if $X = A \cup B$ and A & B are both open then $A=\emptyset$ or $B=\emptyset$ (or both closed)

<u>Thm1</u> A subset X of $\mathbb{R}$ is connected iff it is a "generalized interval". ($\Leftrightarrow \forall a,b \in X, [a,b] \subset X \Leftrightarrow X$ in "convex".)

<u>Thm2</u> A cont. image of a connected set is connected.

<u>Thm3</u> The intermediate-value thm: If X is connected, $f: X \to \mathbb{R}$ is cont., $f(a)<0$ $f(b)>0$, then $\exists x \in X$ s.t. $f(x)=0$.

pre-write

Proof sketches of 1-3.

For 1: Take $X=[0,1]$; assume $A \subset X$ is clopen, $0 \in A$. Let $t_0 = \sup\{t : [0,t] \subset A\}$

1. $t_0 \in A$ 2. if $t<1$, $t_0+\epsilon \in A$ for some ϵ, so $t_0 = 1$. So $t_0 = 1$.

<u>Thm4</u> If A_α are connected and $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

---

done line

<u>Thm</u> The ϵ-nbd theorem: If C is compact and $U \supset C$ is open, then there is some $\epsilon>0$ s.t. $U(C,\epsilon) := \{x : d(C,x) < \epsilon\} \subset U$ where $d(C,x) := \inf_{y \in C} d(x,y)$.

<u>Thm</u> Uniform continuity.