1617-257 Mon Oct 24, Hour 18: Inverse Functions, 3

October 14, 2016 6:25 AM

Riddle Along: Can you pack 125 1x2x4 block in one 10x10x10 cube?
Read Along: Sec 8.
TT Discussion: Wednesday.
Agenda: The Inverse Function Theorem.

Thm (IFT) $f:\mathbb{R}^n \circlearrowleft$ is C^1 near $a\in\mathbb{R}^n$, $\exists\, Df(a)^{-1}$
$\Rightarrow \exists$ nbds $U\ni a$, $V\ni b=f(a)$ s.t. $\exists (f|_U)^{-1}: V\to U$;
$f\in C^r \Rightarrow f|_U^{-1}\in C^r$. WLOG $Df(a)=0$, $a=b=0$.

TL f is Jelly-rigid near a: $\forall \epsilon>0\ \exists$ nbd $J_\epsilon \ni a$ s.t.
$$\forall x,y\in J_\epsilon\ \ \|\underbrace{f(y)-f(x)}_{u}-\underbrace{(y-x)}_{v}\|\leq \epsilon\|y-x\|$$

on board

Part I f is 1-1 on $J_{0.1}$. ($|v|-|u|\leq|u-v|\leq\epsilon|v|$ so $(1-\epsilon)|v|\leq|u|$)

Part II $f|_{J_{0.1}}$ is onto $0.4J_{0.1}$. [Let $U=J_{0.1}\cap f^{-1}(0.4J_{0.1})$ & $V=0.4J_{0.1}$]

should have had Part II.5 : f^{-1} is Jelly-rigid.

Also, part III easier with $\|v\|-\|u\|\leq\|u-v\|\leq\epsilon\|v\| \Rightarrow \|u\|\geq(1-\epsilon)\|v\|$
which is directly cont. of f^{-1} ✓

Part III f^{-1} is cont. on V. (Aside: $|u-v|\leq\epsilon|u|=\epsilon|v+u-v|\leq\epsilon|v|+\epsilon|u-v|$
$(1-\epsilon)|u-v|\leq\epsilon|v|$ so $|u-v|\leq\frac{\epsilon}{1-\epsilon}|v|$)

done line

Part IV f^{-1} is diffable at 0.

Part V f^{-1} is diffable near 0.

Part VI f^{-1} is C^r.