1617-257 Mon Nov 21, Hour 29: Integration Infrastructure

October 14, 2016 6:25 AM

Read Along: Sec 13, 14 (especially important!). Agenda: Some integration and volume infrastructure.
No class no DBN office hours Friday Dec 1! Makeup on Thu Dec 1 5PM at GB120, will be videotaped.
Riddle Along: On R, n ants march in from the left and m march in from the right. Whenever two ants meet, a bang is heard and the two turn backwards and continue marching. How many bangs will be heard? (All speeds always the same).

on board

Chapter 13 in a nutshell

0. If S is a bounded subset of $\mathbb{R}^n$, define $\int_S f := \int_Q f \cdot 1_S$, where $Q \supset S$ is a rectangle. When defined, it is well-defined ☺

1. f, g integrable $\Rightarrow$ $\int_S af+bg = a\int_S f + b\int_S g$ (pf: if $a, b \geq 0$ .... Then $\int(-f) = -\int f$ ...)

2. f, g integrable, $f \leq g$ $\Rightarrow$ $\int_S f \leq \int_S g$

3. f integrable $\Rightarrow$ $\left|\int f\right| \leq \int |f|$

4. $T \subset S$ $\Rightarrow$ $\int_T f \leq \int_S f$

5. $\int_{S_1 \cup S_2} f = \int_{S_1} f + \int_{S_2} f - \int_{S_1 \cap S_2} f$; if $S_1 \cap S_2$ is meas-0, $\int_{S_1 \cup S_2} f = \int_{S_1} f + \int_{S_2} f$.

6. Read the rest ☺

A set is "rectifiable" if 1_S is integrable; the "volume" is then the integral.

Prop. A set is rectifiable iff it is bounded and its boundary is meas-0.

Theorem.

1. v(S)>=0.
2. S_1\subset S_2 implies v(S_1)<=v(S_2).

Done line

3. If S_i are rectifiable, then v(S_1\union S_2)=V(s_1)+v(S_2)-V(intersection).
4. If S is rectifiable, v(S)=0 iff S is of meas-0.
5. If S is rectifiable and f:S->R is bndd cont, than it is integrable.

Not so easy yet we skip: Theorem. If C is compact and rectifiable in R^n, and f,g:C->R are cont with f<=g, Then D={(x,t): x\in C, f(x)<=t<=g(x)} is rectifiable in R^{n+1}, v(D)=\int_C(g-f), and if h is defined on D, then \int_D h = \int_C(\int_f^g h).