Show riddle repository!

HW2 returned, HW3 due, HW4 on web by midnight.

Today's Agenda: Partials commute, the chain rule.

Read Along: sec 6,7.

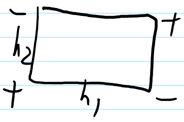
Reminder $f(a+h) = f(a) + Df(a)\cdot h + o(h)$

class C^r: The first r derivatives exist and are cont.

Thm If $f:\mathbb{R}^n \to \mathbb{R}$ is of class C^2, then for every $i \& j$, $\partial_i \partial_j f = \partial_j \partial_i f$

on board

[WLOG, $f:\mathbb{R}^2_{x,y} \to \mathbb{R}$ $a = \binom{x_0}{y_0}$]

Intuition  and why it isn't enough.

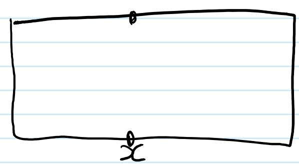
Lemma For any $h>0$, $\exists p,q \in [x_0, x_0+h]\times[y_0,y_0+h]$ s.t.

$$\partial_x \partial_y f(p) = \frac{f(x_0+h, y_0+h) - \ldots + f(x_0,y_0)}{h^2} = \partial_y \partial_x f(q)$$

Lemma $\Rightarrow$ Thm: easy.

Pf of lemma: Call the numerator above λ, set

$g(x) = \dfrac{f(x, y_0+h) - f(x,y_0)}{h}$ Then $\lambda = \dfrac{g(x_0+h) - g(x_0)}{h}$



Then $\exists x_1 \in [x_0, x_0+h]$ s.t.

$$\lambda = \partial_x g(x_1) = \frac{\partial_x f(x_1, y_0+h) - \partial_x f(x, y_0)}{h}$$

and then $\exists y_1 \in [y_0, y_0+h]$ s.t.

$$= \partial_y \partial_x f(x_1, y_1) \Rightarrow q = \binom{x_1}{y_1}. \quad \square$$

done 1/hr

Thm (The chain rule): If $a \in \mathbb{R}^n \xrightarrow{f} \mathbb{R}^m \xrightarrow{g} \mathbb{R}^p$, f diff'able at a, g diff'able at $f(a)$,

then $D(g\circ f)(a) = Dg(f(a))Df(a)$

Example $\mathbb{R}_t \xrightarrow{(t,t)} \mathbb{R}^2_{x,y} \xrightarrow{x^y} \mathbb{R}$

Pf of Thm $\quad f(a+h) = f(a) + F\cdot h + \phi(h) \quad \phi \in o(h)$

$b = f(a) \quad g(b+k) = g(b) + Gk + \gamma(k) \quad \gamma \in o(k)$

$$(g\circ f)(a+h) = g(f(a+h)) = g(\underbrace{f(a)}_{b} + \underbrace{Fh + \phi(h)}_{k}) = g(b) + Gk + \gamma(k)$$

$$= g(f(a)) + GFh + \underbrace{G\cdot\phi(h) + \gamma(Fh + \phi(h))}_{\lambda(h)} = g(f(a)) + GFh + \lambda(h)$$

claim $\lambda(h) \in o(h)$