1617-257 Fri Nov 4, Hour 23: Continuous functions are integrable

October 14, 2016 6:25 AM

TT Discussion at 2:45. Read Along: Secs 11, 1. Riddle Along: V_L=V_S.
Agenda: Continuous functions are integrable

$m_R(f) = \inf_{x\in R} f(x)$ $\quad L(f,P) = \sum_{R\in P} m_R(f)V(R)$ $\quad \underline{\int_Q} f = \sup_P L(f,P)$

$M_R(f) = \sup_{x\in R} f(x)$ $\quad U(f,P) = \sum_{R\in P} M_R(f)V(R)$ $\quad \overline{\int_Q} f = \inf_P U(f,P)$

The Riemann cond.: f is integrable iff $\forall \epsilon>0\ \exists P$ of Q s.t. $U(f,P)-L(f,P)<\epsilon$.

on board.

<u>Example</u> $\int_Q c = c\,\mathrm{Vol}(Q)$

<u>Example</u> If $\lambda(x) = \begin{cases} 1 & x\in\mathbb{Q} \\ 0 & x\notin\mathbb{Q}\end{cases}$ then $\int_{[0,1]}\lambda$ does not exist.

Definition: Uniform continuity.

Examples 1. $y=x$ on $\mathbb{R}$

2. $f(x) = \inf\{d(x,a) : a\in A\}$

3. $y=x^2$ on $\mathbb{R}$.

<u>Thm 2</u> If f is uniformly cont. over Q then it is integrable. done

<u>Thm 1</u> Every cont. function on a compact set is uniformly cont. b.n.

Prob C of HW2:

Problem C. Prove the "Lebesgue number lemma": If $\mathcal{U}=\{U_\alpha\}$ is an open cover of a compact space (X,d), then there exists an $\epsilon>0$ (called "the Lebesgue number of $\mathcal{U}$, such that every open ball of radius ϵ in X is contained in one of the U_α's.