

on board: MAT 475 Problem Solving Seminar
(write a lot! the tricks of the trade!)

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Today's read along: Larson Sec 1.1.

Quiz 1: one of 1.1.6; 1.1.8, 1.1.9.

Today's riddle: Ants Ame, Bela, and Charles (A, B, C) hitch rides on the H, M, S hands of a big floral clock. They start at noon w/ A on H, B on M, and C on S, every time two hands pass each other, the ants on top trade places. They stop at midnight. How many times did each of them go around the center?

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1.1.2. Let $S_{n,0}$, $S_{n,1}$, and $S_{n,2}$ denote the sum of every third element in the n th row of Pascal's Triangle, beginning on the left with the first element, the second element, and the third element respectively. Make a conjecture concerning the value of $S_{100,1}$.

Back to the riddle.

done line

Back to the riddle.

done line

1.1.3. Let x_1, x_2, x_3, \dots be a sequence of nonzero real numbers satisfying

$$x_n = \frac{x_{n-2}x_{n-1}}{2x_{n-2} - x_{n-1}}, \quad n = 3, 4, 5, \dots$$

Establish necessary and sufficient conditions on x_1 and x_2 for x_n to be an integer for infinitely many values of n .

1.1.4. Find positive numbers n and a_1, a_2, \dots, a_n such that $a_1 + \dots + a_n = 1000$ and the product $a_1 a_2 \dots a_n$ is as large as possible.

1.1.5. Let S be a set and $*$ be binary operation on S satisfying the two laws

$$\begin{aligned} x * x &= x && \text{for all } x \text{ in } S, \\ (x * y) * z &= (y * z) * x && \text{for all } x, y, z \text{ in } S. \end{aligned}$$

Show that $x * y = y * x$ for all x, y in S .

$$\begin{aligned} a * b &= \underbrace{(a * b)}_{x * y} \underbrace{(a * b)}_z = \underbrace{(b * (a * b))}_x \underbrace{a}_y \underbrace{z}_z = \underbrace{((a * b) * a)}_b = \underbrace{((b * a) * a)}_b = \\ &\quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \\ &\quad (a * b) * a \quad (a * b) * b \quad (a * a) * b \quad (a * b) * b \\ &\quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \\ &\quad (b * a) * a \quad (b * b) * a \quad (b * b) * a \\ &\quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \quad \parallel \\ &\quad b * a \quad b * a \quad b * a \quad b * a \quad b * a \quad b * a \end{aligned}$$