

Name (Last, First): \_\_\_\_\_

Student ID: \_\_\_\_\_

Dror Bar-Natan: Classes: 2015-16: MAT 475 Problem Solving Seminar:

<http://drorbn.net/16-475>

**Quiz 8** on March 17, 2016: “Divide into Cases”, “Work Backwards”, and the Pigeonhole Principle. You have 25 minutes to solve the following two problems. Please write on both sides of the page. **Good Luck!**

**Problem 1 (near Larson’s 2.5.11).** Let  $T_n$  denote the number of ways of placing  $n$  nonattacking rooks on an  $n \times n$  chessboard so that the resulting arrangement is symmetric about both diagonals. Find a recursive formula for  $T_n$ .

**Problem 2 (near Larson’s 2.6.10).** Let  $\alpha$  be any real number. Prove that among the numbers  $\{\alpha, 2\alpha, \dots, (n-1)\alpha\}$  there is one that differs from an integer by at most  $1/n$ . *Hint.* The bins could be  $[\frac{1}{n}, \frac{2}{n}), [\frac{2}{n}, \frac{3}{n}), \dots, [\frac{n-2}{n}, \frac{n-1}{n})$ .