

Name (Last, First): _____

Student ID: _____

Dror Bar-Natan: Classes: 2015-16: MAT 475 Problem Solving Seminar:

<http://drorbn.net/16-475>

Quiz 4 on February 4, 2016: “Modify the Problem” / “Choose an Effective Notation”. You have 30 minutes to solve as much as you can of the problems below. Please write on both sides of the page. **Good Luck!**

Problem 1 (Larson’s 1.4.3). Prove that there does not exist positive integers x , y , and z such that $x^2 + y^2 + z^2 = 2xyz$.

Problem 2 (Larson’s 1.5.10). A well known theorem asserts that a prime $p > 2$ can be written as a sum of two perfect squares ($p = m^2 + n^2$ with m and n integers) iff p is 1 mod 4. Assuming this, prove:

1. Every prime which is 1 mod 8 can be written as $x^2 + 16y^2$, with x and y integers.
2. Every prime which is 5 mod 8 can be written as $(2x + y)^2 + 4y^2$, with x and y integers.

Problem 3 (no credit, yet the best solutions will be advertised). What is your favourite “Modify the Problem” or “Choose an Effective Notation” problem?