

Pensieve header: Two Peano curves and Non-Commutative Gaussian Elimination, Day 4.

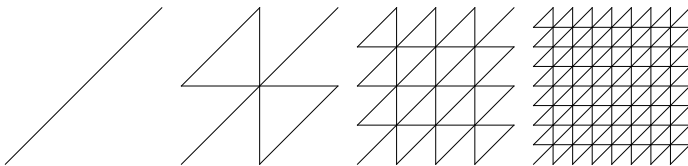
From <http://www.math.toronto.edu/drorbn/classes/16-1750-ShamelessMathematica/About.html>: **Possible Topics** (in no particular order). Whatever you may suggest, and the **Fibonacci numbers**; **the Jones polynomial**; **a more efficient Jones algorithm**; **a riddle on spheres**; **Khovanov homology**; Γ -calculus; the Hopf fibration; **Hilbert's 13th problem**; **non-commutative Gaussian elimination**; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; **an order 4 torus**; **the Schwarz Lantern**; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; **a Peano curve**; braid closures and Vogel's algorithm; **the insolubility of the quintic**; **phase portraits**; **the Mandelbrot set**.

A Peano Curve

```

Peano[0] = DLine[{{0, 0}, {1, 1}}];
Peano[n_] := Peano[n - 1] /. DLine[{{x0_, y0_}, {x1_, y1_}}] => {
  dx = x1 - x0; dy = y1 - y0;
  DLine[{{x0, y0}, {x0 + 1/2 dx, y0 + 1/2 dy}}],
  Line[{{x0 + 1/2 dx, y0 + 1/2 dy}, {x0 + 1/2 dx, y0}}],
  DLine[{{x0 + 1/2 dx, y0}, {x0 + dx, y0 + 1/2 dy}}],
  Line[{{x0 + dx, y0 + 1/2 dy}, {x0, y0 + 1/2 dy}}],
  DLine[{{x0, y0 + 1/2 dy}, {x0 + 1/2 dx, y0 + dy}}],
  Line[{{x0 + 1/2 dx, y0 + dy}, {x0 + 1/2 dx, y0 + 1/2 dy}}],
  DLine[{{x0 + 1/2 dx, y0 + 1/2 dy}, {x0 + dx, y0 + dy}}]
};
GraphicsRow@Table[Graphics[Peano[n] /. DLine -> Line], {n, 0, 3}]

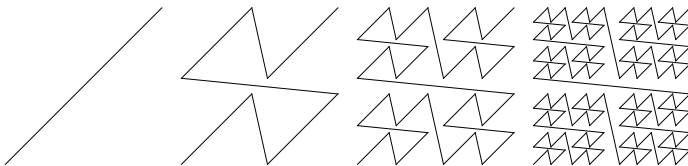
```



```

AlmostPeano[0] = DLine[{{0, 0}, {1, 1}}];
AlmostPeano[n_] := AlmostPeano[n - 1] /. DLine[{{x0_, y0_}, {x1_, y1_}}] => {
  dx = x1 - x0; dy = y1 - y0;
  DLine[{{x0, y0}, {x0 + 0.45 dx, y0 + 0.45 dy}}],
  Line[{{x0 + 0.45 dx, y0 + 0.45 dy}, {x0 + 0.55 dx, y0}}],
  DLine[{{x0 + 0.55 dx, y0}, {x0 + dx, y0 + 0.45 dy}}],
  Line[{{x0 + dx, y0 + 0.45 dy}, {x0, y0 + 0.55 dy}}],
  DLine[{{x0, y0 + 0.55 dy}, {x0 + 0.45 dx, y0 + dy}}],
  Line[{{x0 + 0.45 dx, y0 + dy}, {x0 + 0.55 dx, y0 + 0.55 dy}}],
  DLine[{{x0 + 0.55 dx, y0 + 0.55 dy}, {x0 + dx, y0 + dy}}]
};
GraphicsRow@Table[Graphics[AlmostPeano[n] /. DLine -> Line], {n, 0, 3}]

```



`Graphics[AlmostPeano[5] /. DLine -> Line]`



A Better Peano Curve

`t = RandomChoice[{0, 1}, 20]`

`{1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0}`

`2 t`

`{2, 2, 0, 2, 2, 0, 0, 0, 2, 0, 2, 2, 0, 0, 2, 0, 2, 0, 2, 0}`

`2 t. (1/3)Range@20 // N`

`0.921928`

`2 t. (1/3)Range@20`

`1 071 520 832`

`1 162 261 467`

`FromDigits[{2 t, 0}, 3]`

`1 071 520 832`

`1 162 261 467`

`t`

`{1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0}`

`Partition[t, 2]`

`{{1, 1}, {0, 1}, {1, 0}, {0, 0}, {1, 0}, {1, 1}, {0, 0}, {1, 0}, {1, 0}, {1, 0}}`

```

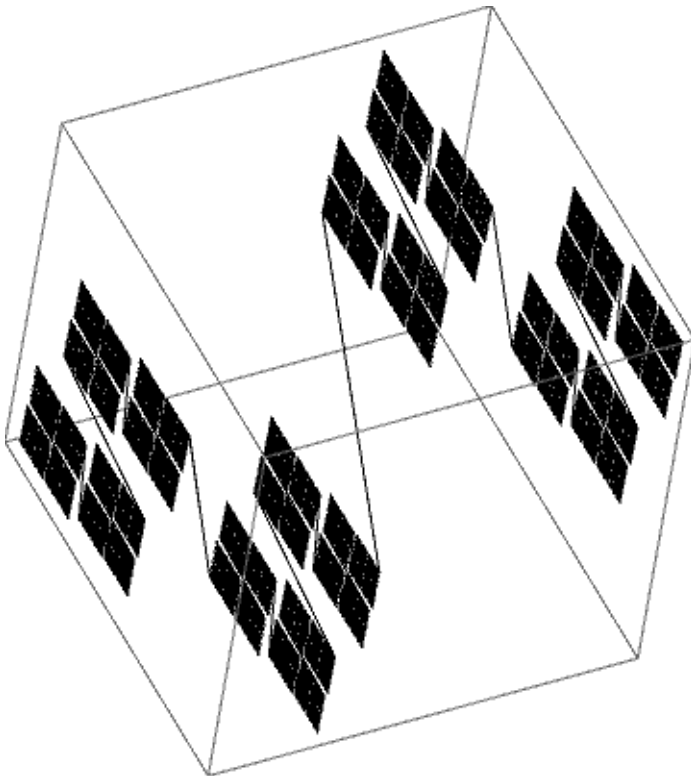
Partition[t, 2] // Transpose
{{1, 0, 1, 0, 1, 1, 0, 1, 1, 1}, {1, 1, 0, 0, 0, 1, 0, 0, 0, 0}}

Prepend[
  FromDigits[{-#, 0}, 2] & /@ Transpose[Partition[t, 2]],
  FromDigits[{2 t, 0}, 3]
]
{
 $\frac{1\ 071\ 520\ 832}{1\ 162\ 261\ 467}$ ,  $\frac{695}{1024}$ ,  $\frac{49}{64}$ 
}

γ[t_List] := Prepend[
  FromDigits[{-#, 0}, 2] & /@ Transpose[Partition[t, 2]],
  FromDigits[{2 t, 0}, 3]
];

Graphics3D[
  Line[γ /@ Tuples[{0, 1}, 14]],
  ViewPoint → {-370.06, 733.124, 570.6},
  ViewVertical → {0.1792, 0.789508, -0.599889}
] // Rasterize

```



Non Commutative Gaussian Elimination

```

n = 54;
g1 = Cycles[{{1, 18, 45, 28}, {2, 27, 44, 19},
            {3, 36, 43, 10}, {46, 52, 54, 48}, {47, 49, 53, 51}}];
g2 = Cycles[{{7, 16, 39, 30}, {8, 25, 38, 21}, {9, 34, 37, 12},
            {13, 15, 33, 31}, {14, 24, 32, 22}}];
g3 = Cycles[{{28, 31, 34, 48}, {29, 32, 35, 47}, {30, 33, 36, 46},
            {37, 39, 45, 43}, {38, 42, 44, 40}}];
g4 = Cycles[{{1, 3, 9, 7}, {2, 6, 8, 4}, {10, 54, 16, 13},
            {11, 53, 17, 14}, {12, 52, 18, 15}}];
g5 = Cycles[{{1, 13, 37, 46}, {4, 22, 40, 49}, {7, 31, 43, 52},
            {10, 12, 30, 28}, {11, 21, 29, 19}}];
g6 = Cycles[{{3, 48, 39, 15}, {6, 51, 42, 24}, {9, 54, 45, 33},
            {16, 18, 36, 34}, {17, 27, 35, 25}}];

σ_ ∘ τ_ := PermutationProduct[τ, σ];
Feed[Cycles[{}]] := 1 + 1;
Feed[τ_] := Module[{i, j, k, l},
  i = Min[PermutationSupport[τ]];
  j = PermutationReplace[i, τ];
  If[Head[σi,j] === Cycles,
    Feed[InversePermutation[σi,j] ∘ τ],
    (*Else*) σi,j = τ;
  For[k = 1, k < n, ++k,
    For[l = k + 1, l ≤ n, ++l,
      If[Head[σk,l] === Cycles,
        Feed[σi,j ∘ σk,l]; Feed[σk,l ∘ σi,j]]
    ]
  ];
$RecursionLimit = ∞;

Table[Feed[gα],
  {α, 6}] // Timing

$$\prod_{i=1}^n (1 + \text{Count}[\text{Range}[n], j_ /; \text{Head}[\sigma_{i,j}] = \text{Cycles}]), \{\alpha, 6\} // \text{Timing}$$


```

Mathematica Challenges.

1. Can you find the true lengths of the “tricks”? Done!
2. Can you fix these sad results? Done, but can you do better?
3. Can you turn the program into a “package”?
4. Can you make the program doubly efficient?
5. Can you keep track of the actual “tricks”?
6. Can you actually solve cubes?

The Mathematica Package “Permutations”

Goal: Re-implement permutations, using “permutation lists” as the basic data type, rather than “cycle decompositions”.

? Cycles

Cycles[{*cyc*₁, *cyc*₂, ...}] represents a permutation with disjoint cycles *cyc*_{*i*}. >>

? PermutationList

PermutationList[*perm*] returns a permutation list representation of permutation *perm*.

PermutationList[*perm*, *len*] returns a permutation list of length *len*. >>

? PermutationCycles

PermutationCycles[*perm*] gives a disjoint cycle representation of permutation *perm*. >>

? PermutationProduct

PermutationProduct[*a*, *b*, *c*] gives the product of permutations *a*, *b*, *c*. >>

? InversePermutation

InversePermutation[*perm*] returns the inverse of permutation *perm*. >>

? PermutationSupport

PermutationSupport[*perm*] returns the support of the permutation *perm*. >>

? PermutationReplace

PermutationReplace[*expr*, *perm*] replaces each part in *expr* by its image under the permutation *perm*.

PermutationReplace[*expr*, *gr*] returns the list of images of *expr* under all elements of the permutation group *gr*. >>

Finer Goal: Make a useful package implementing PL, PC, PP, IP, PS, and PR, to the point that we can re-write “Feed” so that it works on the following γ_α 's:

Table[$\gamma_\alpha = \text{PermutationList}[\mathbf{g}_\alpha, \mathbf{n}]$, { α , 6}] // **MatrixForm**

18	27	36	4	5	6	7	8	9	3	11	12	13	14	15	16	17	45	2	20	21	22	23	24	25	:
1	2	3	4	5	6	16	25	34	10	11	9	15	24	33	39	17	18	19	20	8	14	23	32	38	:
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	:
3	6	9	2	5	8	1	4	7	54	53	52	10	11	12	13	14	15	19	20	21	22	23	24	25	:
13	2	3	22	5	6	31	8	9	12	21	30	37	14	15	16	17	18	11	20	29	40	23	24	25	:
1	2	48	4	5	51	7	8	54	10	11	12	13	14	3	18	27	36	19	20	21	22	23	6	17	:

? Permutation

Information::notfound : Symbol Permutation not found. >>