Pensieve header: Implementing KH, day 2.

Road map:

1. Produce all the info in a “cube of smoothings”.
2. Produce a commuting cube of vector spaces and maps between them.
3. Produce a complex.
4. Compute homology.

```
K[3, 1] = Xm[1, 4, 2, 5] Xm[3, 6, 4, 1] Xm[5, 2, 6, 3];
K[5, 1] = Xm[1, 6, 2, 7] Xm[3, 8, 4, 9] Xm[5, 10, 6, 1] Xm[7, 2, 8, 3] Xm[9, 4, 10, 5];
K[10, 132] =
  Xm[5, 12, 6, 13] Xm[9, 16, 10, 17] Xm[11, 6, 12, 7] Xm[13, 20, 14, 1] Xm[15, 18, 16, 19]
   Xm[17, 10, 18, 11] Xm[19, 14, 20, 15] Xp[2, 8, 3, 7] Xp[4, 2, 5, 1] Xp[8, 4, 9, 3];
```

```
SetAttributes[p, Orderless];
SetAttributes[dx, Orderless];
α = 0;
ϵ /: ϵ^n_ /; n > 1 = 0;
t1 = Expand[
      dx[] K[3, 1] /. {
          (Xp | Xm)[i_, j_, k_, l_] :> (++α;
            p[i, j] p[k, l] + dx[α] p[i, l] p[j, k] + ϵ e[α] X[i, j, k, l])
        } /. p[i_, j_] :> p[i, j][Min[i, j]]
     ] //. p[i_, j_][m_] p[j_, k_][n_] :> p[i, k][Min[m, n]] /. {
     X[i_, j_, k_, l_] p[i_, j_][m_] p[k_, l_][n_] :> (c[m] c[n] → c[Min[m, n]]),
     X[i_, j_, k_, l_] p[i_, l_][m_] p[j_, k_][n_] :> (c[Min[m, n]] → c[m] c[n])
    } /.
   p[i_, j_][m_]-⋅ :> c[m] //. dx[i___] dx[j___] :> dx[i, j]
```

```
c[1] c[2] c[3] dx[] + c[1] c[3] dx[1] + c[1] c[2] dx[2] + c[1] c[2] dx[3] +
 c[1] dx[1, 2] + c[1] dx[1, 3] + c[1] dx[2, 3] + c[1] c[2] dx[1, 2, 3] +
 ϵ dx[2, 3] e[1] (c[1] → c[1] c[2]) + ϵ dx[1, 3] e[2] (c[1] → c[1] c[2]) +
 ϵ dx[1, 2] e[3] (c[1] → c[1] c[2]) + ϵ c[3] dx[] e[1] (c[1] c[2] → c[1]) +
 ϵ dx[2] e[1] (c[1] c[2] → c[1]) + ϵ dx[3] e[1] (c[1] c[2] → c[1]) +
 ϵ dx[3] e[2] (c[1] c[2] → c[1]) + ϵ dx[2] e[3] (c[1] c[2] → c[1]) +
 ϵ c[2] dx[] e[2] (c[1] c[3] → c[1]) + ϵ dx[1] e[2] (c[1] c[3] → c[1]) +
 ϵ dx[1] e[3] (c[1] c[3] → c[1]) + ϵ c[1] dx[] e[3] (c[2] c[3] → c[2])
```

3^{10}

59 049

```
t1 /. ϵ → 0
```

```
c[1] c[2] c[3] dx[] + c[1] c[3] dx[1] + c[1] c[2] dx[2] + c[1] c[2] dx[3] +
 c[1] dx[1, 2] + c[1] dx[1, 3] + c[1] dx[2, 3] + c[1] c[2] dx[1, 2, 3]
```

```
Coefficient[t1, ϵ]
```

```
dx[2, 3] e[1] (c[1] → c[1] c[2]) + dx[1, 3] e[2] (c[1] → c[1] c[2]) +
 dx[1, 2] e[3] (c[1] → c[1] c[2]) + c[3] dx[] e[1] (c[1] c[2] → c[1]) +
 dx[2] e[1] (c[1] c[2] → c[1]) + dx[3] e[1] (c[1] c[2] → c[1]) +
 dx[3] e[2] (c[1] c[2] → c[1]) + dx[2] e[3] (c[1] c[2] → c[1]) +
 c[2] dx[] e[2] (c[1] c[3] → c[1]) + dx[1] e[2] (c[1] c[3] → c[1]) +
 dx[1] e[3] (c[1] c[3] → c[1]) + c[1] dx[] e[3] (c[2] c[3] → c[2])
```

```
t1 /. c[i_] :> (q + 1/q)
```

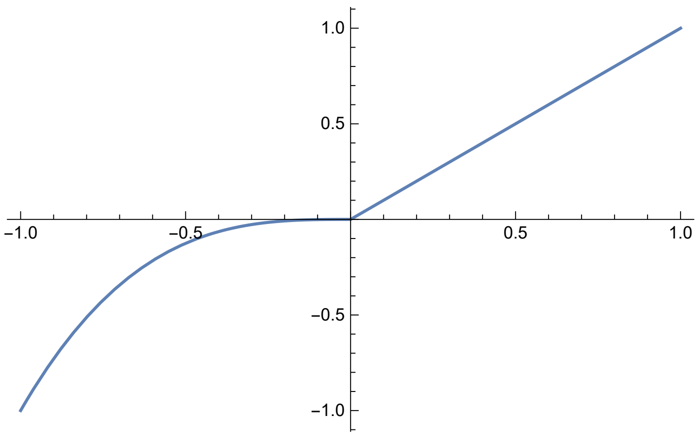
$$3 A B^2 \left(\frac{1}{q}+q\right)+3 A^2 B \left(\frac{1}{q}+q\right)^2+B^3 \left(\frac{1}{q}+q\right)^2+A^3 \left(\frac{1}{q}+q\right)^3$$

```
log /: log[a_] + log[b_] := log[a b]
```

```
log[7] + log[5]
```

```
log[35]
```

```
f[x_] /; x > 0 := x;
f[x_] /; x < 0 := x^3;
Plot[f[x], {x, -1, 1}]
```




```
{a;}
```

```
{Null}
```

```
S_{a__}[K_] := Table[
   {{a}[[i]], K[[i]]}, {i, 1, Length[K]}
  ] /. {
   {0, Xp[i_, j_, k_, l_] :> ...},
   {1, Xp} :>  ..
      {0, Xm} :> ...
```