

Pensieve header: February 8: A faster Jones program.

The old Jones program

```

SetAttributes[P, Orderless];
OldJones[Knot_] := Module[{t1, t2},
  t1 = Expand[Knot /. {
    Xp[i_, j_, k_, l_] := q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
    Xm[i_, j_, k_, l_] := -q^-2 * P[i, j] P[k, l] + q^-1 * P[i, l] P[j, k]
  }];
  t2 = t1 //. P[a_, b_] P[b_, c_] := P[a, c];
  Simplify[t2 /. {P[i_, i_] := (q + 1/q), P[i_, j_]^2 := (q + 1/q)}]
]

```

A 48-crossing knot

```

K = Xp[4, 88, 5, 87] Xp[5, 75, 6, 74] Xp[6, 62, 7, 61] Xp[7, 49, 8, 48]
  Xp[8, 36, 9, 35] Xp[9, 23, 10, 22] Xp[16, 4, 17, 3] Xp[17, 87, 18, 86]
  Xp[18, 74, 19, 73] Xp[19, 61, 20, 60] Xp[20, 48, 21, 47] Xp[21, 35, 22, 34]
  Xp[28, 16, 29, 15] Xp[29, 3, 30, 2] Xp[30, 86, 31, 85] Xp[31, 73, 32, 72]
  Xp[32, 60, 33, 59] Xp[33, 47, 34, 46] Xp[40, 28, 41, 27] Xp[41, 15, 42, 14]
  Xp[42, 2, 43, 1] Xp[43, 85, 44, 84] Xp[44, 72, 45, 71] Xp[45, 59, 46, 58]
  Xp[52, 40, 53, 39] Xp[53, 27, 54, 26] Xp[54, 14, 55, 13] Xp[55, 1, 56, 96]
  Xp[56, 84, 57, 83] Xp[57, 71, 58, 70] Xp[64, 52, 65, 51] Xp[65, 39, 66, 38]
  Xp[66, 26, 67, 25] Xp[67, 13, 68, 12] Xp[68, 96, 69, 95] Xp[69, 83, 70, 82]
  Xp[76, 64, 77, 63] Xp[77, 51, 78, 50] Xp[78, 38, 79, 37] Xp[79, 25, 80, 24]
  Xp[80, 12, 81, 11] Xp[81, 95, 82, 94] Xp[88, 76, 89, 75] Xp[89, 63, 90, 62]
  Xp[90, 50, 91, 49] Xp[91, 37, 92, 36] Xp[92, 24, 93, 23] Xp[93, 11, 94, 10];

```

The new Jones program

```

Dynamic[todo1]
todo1

1 + 1
2

```

```

NewJones[K_] := Module[{J = 1, todo = List@@K, inter = {}, x},
  While[Length[todo] ≥ 1,
    todo1 = todo;
    x = RandomChoice[MaximalBy[todo, Length[inter ∩ (List@@#)] &]];
    J = Expand[J * (x /. {
      Xp[i_, j_, k_, l_] => q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
      Xm[i_, j_, k_, l_] => -q^-2 * P[i, j] P[k, l] + q^-1 * P[i, l] P[j, k]
    })] /. P[a_, b_] P[b_, c_] => P[a, c] /.
    {P[i_, i_] => (q+1/q), P[i_, j_]^2 => (q+1/q)};
    inter = Complement[inter ∪ List@@x, inter ∩ List@@x];
    todo = Complement[todo, {x}]
  ];
  Expand[J]
]

```

Jones[K]

$$q^{41} + q^{43} + q^{45} + q^{47} + q^{49} + q^{51} + q^{53} + q^{55} - q^{59} - q^{61} - q^{63} - q^{65} - q^{67} - q^{69}$$

```
inter = Complement[inter ∪ List@@x, inter ∩ List@@x]
```

```
{32, 33, 59, 60}
```

```

J = Expand[J * (x /. {
  Xp[i_, j_, k_, l_] => q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
  Xm[i_, j_, k_, l_] => -q^-2 * P[i, j] P[k, l] + q^-1 * P[i, l] P[j, k]
})] /. P[a_, b_] P[b_, c_] => P[a, c] /.
{P[i_, i_] => (q+1/q), P[i_, j_]^2 => (q+1/q)}

```

$$q P[32, 60] P[33, 59] - q^2 P[32, 59] P[33, 60]$$

```
{J = 1, todo = List@@K, inter = {}}
```

```

{1, {Xp[4, 88, 5, 87], Xp[5, 75, 6, 74], Xp[6, 62, 7, 61], Xp[7, 49, 8, 48],
  Xp[8, 36, 9, 35], Xp[9, 23, 10, 22], Xp[16, 4, 17, 3], Xp[17, 87, 18, 86],
  Xp[18, 74, 19, 73], Xp[19, 61, 20, 60], Xp[20, 48, 21, 47], Xp[21, 35, 22, 34],
  Xp[28, 16, 29, 15], Xp[29, 3, 30, 2], Xp[30, 86, 31, 85], Xp[31, 73, 32, 72],
  Xp[32, 60, 33, 59], Xp[33, 47, 34, 46], Xp[40, 28, 41, 27], Xp[41, 15, 42, 14],
  Xp[42, 2, 43, 1], Xp[43, 85, 44, 84], Xp[44, 72, 45, 71], Xp[45, 59, 46, 58],
  Xp[52, 40, 53, 39], Xp[53, 27, 54, 26], Xp[54, 14, 55, 13], Xp[55, 1, 56, 96],
  Xp[56, 84, 57, 83], Xp[57, 71, 58, 70], Xp[64, 52, 65, 51], Xp[65, 39, 66, 38],
  Xp[66, 26, 67, 25], Xp[67, 13, 68, 12], Xp[68, 96, 69, 95], Xp[69, 83, 70, 82],
  Xp[76, 64, 77, 63], Xp[77, 51, 78, 50], Xp[78, 38, 79, 37], Xp[79, 25, 80, 24],
  Xp[80, 12, 81, 11], Xp[81, 95, 82, 94], Xp[88, 76, 89, 75], Xp[89, 63, 90, 62],
  Xp[90, 50, 91, 49], Xp[91, 37, 92, 36], Xp[92, 24, 93, 23], Xp[93, 11, 94, 10]}, {}
}

```

