Pensieve header: January 25: Fibonacci Forever.

```
f1[0] = 1; f1[1] = 1;
f1[n_] := (f1[n] = f1[n - 1] + f1[n - 2])
```

```
Table[f1[k], {k, 1, 10}]
```

```
{1, 2, 3, 5, 8, 13, 21, 34, 55, 89}
```

```
f1[100]
```

```
573 147 844 013 817 084 101
```

```
f2[n_] := Module[{k = 1, prev, cur},
  prev = cur = 1;
  While[k < n, {prev, cur} = {cur, prev + cur}; ++k];
  cur
 ]
```

```
f2[10]
```

```
89
```

```
f2[100]
```

```
573 147 844 013 817 084 101
```

```
? For
```

For[*start*, *test*, *incr*, *body*] executes *start*, then repeatedly evaluates *body* and *incr* until *test* fails to give True. >>

```
f3[n_] := (For[k = cur = prev = 1, k < n, k += 1, {prev, cur} = {cur, prev + cur}]; cur)
```

```
f3[100]
```

```
573 147 844 013 817 084 101
```

```
? Do
```

Do[*expr*, *n*] evaluates *expr* *n* times.

Do[*expr*, {*i*, i_{max}}] evaluates *expr* with the variable *i* successively taking on the values 1 through i_{max} (in steps of 1).

Do[*expr*, {*i*, i_{min}, i_{max}}] starts with $i = i_{min}$.

Do[*expr*, {*i*, i_{min}, i_{max}, *di*}] uses steps *di*.

Do[*expr*, {*i*, {i_1, i_2, …}}] uses the successive values i_1, i_2, ….

Do[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, …] evaluates *expr* looping over different values of *j* etc. for each *i*. >>

```
f4[n_] := (
  prev = cur = 1;
  Do[{prev, cur} = {cur, prev + cur}, n - 1];
  cur)
```

```
f4[100]
```

573 147 844 013 817 084 101

```
f4[2]
```

2

```
f5[n_] := {1, 1, 1} //. {
    {n, _, c_} :> c,
    {k_, p_, c_} :> {k + 1, c, p + c}
  }
```

```
f5[100]
```

573 147 844 013 817 084 101

```
? //.
```

expr //. *rules* repeatedly performs replacements until *expr* no longer changes. ≫

$$\text{Series}\left[\frac{1}{1-x-x^2}, \{x, 0, 10\}\right]$$

$1 + x + 2 x^2 + 3 x^3 + 5 x^4 + 8 x^5 + 13 x^6 + 21 x^7 + 34 x^8 + 55 x^9 + 89 x^{10} + O[x]^{11}$

$$\text{Series}\left[\frac{1}{1-x-x^2}, \{x, 0, 10\}\right] \text{ // FullForm}$$

```
SeriesData[x, 0, List[1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89], 0, 11, 1]
```

$$\text{Series}\left[\frac{1}{1-x-x^2}, \{x, 0, 100\}\right][[3]] \text{ // Last}$$

573 147 844 013 817 084 101

$$\text{SeriesCoefficient}\left[\frac{1}{1-x-x^2}, \{x, 0, 100\}\right]$$

573 147 844 013 817 084 101

$$\frac{\text{D}\left[\frac{1}{1-x-x^2}, \{x, 100\}\right]}{100!} \text{ /. } x \to 0$$

573 147 844 013 817 084 101

```
f8[n_] := Sum[Binomial[n - k, k], {k, 0, Floor[n/2]}]
```

```
f8[100]
```

573 147 844 013 817 084 101

```
M = {{0, 1}, {1, 1}}
```

{{0, 1}, {1, 1}}

```
M // MatrixForm
```

$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

```
?MatrixPower
```

MatrixPower[m, n] gives the n^{th} matrix power of the matrix m.

MatrixPower[m, n, v] gives the n^{th} matrix power of the matrix m applied to the vector v. >>

MatrixPower$\left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, 100\right]$[[2, 2]]

573 147 844 013 817 084 101

MatrixPower$\left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, 100\right]$⟦2, 2⟧

573 147 844 013 817 084 101

```
Eigensystem[M]
```

$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \left\{\left\{\frac{1}{2}\left(-1+\sqrt{5}\right), 1\right\}, \left\{\frac{1}{2}\left(-1-\sqrt{5}\right), 1\right\}\right\}\right\}$

2×3

6

```
2.3
```

2.3

$2. \times 3$

6.

Solve$\left[\lambda^2 == \lambda + 1, \lambda\right]$

$\left\{\left\{\lambda \to \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \left\{\lambda \to \frac{1}{2}\left(1+\sqrt{5}\right)\right\}\right\}$

{λ1, λ2} = λ /. Solve$\left[\lambda^2 == \lambda + 1, \lambda\right]$

$\left\{\frac{1}{2}\left(1-\sqrt{5}\right), \frac{1}{2}\left(1+\sqrt{5}\right)\right\}$

f10[n_] := $\alpha\, \lambda1^n + \beta\, \lambda2^n$

```
f10[0]
```

$\alpha + \beta$

```
f10[1]
```

$\frac{1}{2}\left(1-\sqrt{5}\right)\alpha + \frac{1}{2}\left(1+\sqrt{5}\right)\beta$

```
Clear[α, β]
```

```
Solve[{f10[0] == 1, f10[1] == 1}, {α, β}]
```

$\left\{\left\{\alpha \to \frac{1}{10}\left(5-\sqrt{5}\right), \beta \to \frac{1}{10}\left(5+\sqrt{5}\right)\right\}\right\}$

```
Solve[{f10[0] == 1, f10[1] == 1}, {α, β}] /. Rule → Set
```

$\left\{\left\{\frac{1}{10}\left(5-\sqrt{5}\right), \frac{1}{10}\left(5+\sqrt{5}\right)\right\}\right\}$

α

α

f10[100] /. $\left\{\alpha \to \frac{1}{10}\left(5-\sqrt{5}\right), \beta \to \frac{1}{10}\left(5+\sqrt{5}\right)\right\}$ **// Expand**

573 147 844 013 817 084 101

α

α

Expand$\left[(x+y)^{12}\right]$

$x^{12} + 12\, x^{11}\, y + 66\, x^{10}\, y^2 + 220\, x^9\, y^3 + 495\, x^8\, y^4 + 792\, x^7\, y^5 + 924\, x^6\, y^6 + 792\, x^5\, y^7 + 495\, x^4\, y^8 + 220\, x^3\, y^9 + 66\, x^2\, y^{10} + 12\, x\, y^{11} + y^{12}$

```
%56 // Factor
```

$(x+y)^{12}$