

Pensieve header: January 18: Solving the MAT 475 Quiz 1 on the computer.

```
bp[n_, 0] = bp[n_, n_] = 1;
bp[n_, k_] := bp[n, k] = bp[n-1, k-1] + bp[n-1, k] /. 2 -> 0
```

```
Sum[bp[1000, k], {k, 0, 1000}]
```

64

$$\sum_{k=0}^{1000} bp[1000, k]$$

64

```
tab = Table[bp[n, k], {n, 0, 10}, {k, 0, n}]
```

```
{{1}, {1, 1}, {1, 0, 1}, {1, 1, 1, 1}, {1, 0, 0, 0, 1}, {1, 1, 0, 0, 1, 1},
 {1, 0, 1, 0, 1, 0, 1}, {1, 1, 1, 1, 1, 1, 1}, {1, 0, 0, 0, 0, 0, 0, 1},
 {1, 1, 0, 0, 0, 0, 0, 0, 1, 1}, {1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1}}
```

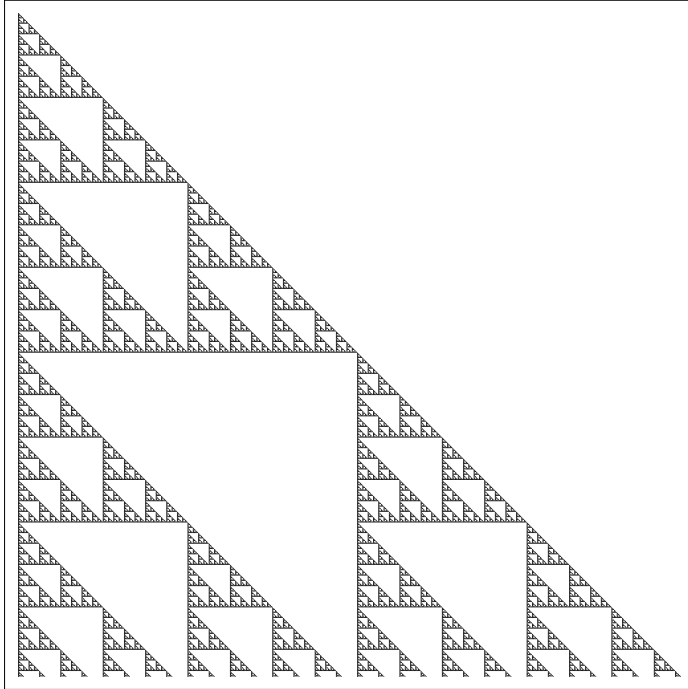
```
tab // MatrixForm
```

```
(
      {1}
     {1, 1}
    {1, 0, 1}
   {1, 1, 1, 1}
  {1, 0, 0, 0, 1}
 {1, 1, 0, 0, 1, 1}
{1, 0, 1, 0, 1, 0, 1}
{1, 1, 1, 1, 1, 1, 1}
{1, 0, 0, 0, 0, 0, 0, 1}
{1, 1, 0, 0, 0, 0, 0, 0, 1, 1}
{1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1}
)
```

? ArrayPlot

ArrayPlot[array] generates a plot in which the values in an array are shown in a discrete array of squares. >>

```
ArrayPlot[Table[bp[n, k], {n, 0, 1000}, {k, 0, n}]]
```



```
s[0] = 1; s[n_] := s[n] = 2 s[n - 2Log[2, n]];
```

```
s[1000]
```

64

```
s[106]
```

128

```
? s
```

```
Global`S
```

```
S[0] = 1
S[8] = 2
S[40] = 4
S[64] = 2
S[104] = 8
S[232] = 16
S[488] = 32
S[576] = 4
S[1000] = 64
S[16960] = 8
S[82496] = 16
S[213568] = 32
S[475712] = 64
S[1000000] = 128
S[n_] := S[n] = 2 S[n - 2Floor[Log[2,n]]]
```

```
{1, 2, 4, 5, 8}
```

```
{1, 2, 4, 5, 8}
```

```
"12458"
```

```
12458
```

```
sl[1] = {"", "1"};
sl[n_] := sl[n] = Join[
  sl[n-1],
  (#<> ToString[n]) & /@ Reverse[sl[n-1]]
]
```

```
sl[2] /. "" -> "∅"
```

```
{∅, 1, 12, 2}
```

```
sl[3] /. "" -> "∅"
```

```
{∅, 1, 12, 2, 23, 123, 13, 3}
```

```
sl[5] /. "" -> "∅"
```

```
{∅, 1, 12, 2, 23, 123, 13, 3, 34, 134, 1234, 234, 24, 124, 14, 4, 45, 145,
  1245, 245, 2345, 12345, 1345, 345, 35, 135, 1235, 235, 25, 125, 15, 5}
```

```
sl[10] /. "" -> "∅"
```

$\{\emptyset, 1, 12, 2, 23, 123, 13, 3, 34, 134, 1234, 234, 24, 124, 14, 4, 45, 145, 1245, 245, 2345,$   
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```

**?? Times**

$x*y*z$ ,  $x*y^*z$ , or  $xyz$  represents a product of terms. >>

```
Attributes[Times] = {Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}
```

```
Default[Times] := 1
```

```
2 {1, 2, 3}
```

```
{2, 4, 6}
```

**?? StringJoin**

" $s_1$ " <> " $s_2$ " <> ..., `StringJoin[" $s_1$ ", " $s_2$ ", ...]`, or `StringJoin[{" $s_1$ ", " $s_2$ ", ...}]` yields a string consisting of a concatenation of the  $s_i$ . >>

```
Attributes[StringJoin] = {Flat, OneIdentity, Protected}
```

```
SetAttributes[StringJoin, Listable]
```

```
Unprotect[Plus];
```

```
a_ + b_ := a * b;
```

```
Protect[Plus]
```

```
{Plus}
```

```
4 + 5
```

```
9
```

```
Unprotect[Plus]
```

```
{Plus}
```

```
Clear[Plus]
```

```
2 + 3
```

```
5
```