

Pensieve header: January 11: Computing the Jones polynomial, rough procedure.

t1 = Xp[1, 5, 2, 4] Xp[5, 3, 6, 2] Xp[3, 1, 4, 6]

Xp[1, 5, 2, 4] Xp[3, 1, 4, 6] Xp[5, 3, 6, 2]

t1

Xp[1, 5, 2, 4] Xp[3, 1, 4, 6] Xp[5, 3, 6, 2]

t2 = t1 /. Xp[i_, j_, k_, l_] => q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k]

$(-q^2 P[1, 4] P[3, 6] + q P[3, 1] P[4, 6])$

$(q P[1, 5] P[2, 4] - q^2 P[1, 4] P[5, 2]) (-q^2 P[3, 6] P[5, 2] + q P[5, 3] P[6, 2])$

t3 = Expand[t2]

$q^5 P[1, 4] P[1, 5] P[2, 4] P[3, 6]^2 P[5, 2] -$

$q^4 P[1, 5] P[2, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2] -$

$q^6 P[1, 4]^2 P[3, 6]^2 P[5, 2]^2 + q^5 P[1, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2]^2 -$

$q^4 P[1, 4] P[1, 5] P[2, 4] P[3, 6] P[5, 3] P[6, 2] +$

$q^3 P[1, 5] P[2, 4] P[3, 1] P[4, 6] P[5, 3] P[6, 2] +$

$q^5 P[1, 4]^2 P[3, 6] P[5, 2] P[5, 3] P[6, 2] -$

$q^4 P[1, 4] P[3, 1] P[4, 6] P[5, 2] P[5, 3] P[6, 2]$

t4 = t3 //. P[a_, b_] P[b_, c_] => P[a, c]

$-q^4 P[1, 4]^2 + q^3 P[2, 2] P[3, 3] - q^4 P[3, 6]^2 + q^5 P[1, 4]^2 P[3, 6]^2 - q^4 P[5, 2]^2 +$

$q^5 P[1, 4]^2 P[5, 2]^2 + q^5 P[3, 6]^2 P[5, 2]^2 - q^6 P[1, 4]^2 P[3, 6]^2 P[5, 2]^2$

t5 = t4 /. {P[i_, i_] => (q+1/q), P[i_, j_]^2 => (q+1/q)}

$-3 q^4 \left(\frac{1}{q} + q\right) + q^3 \left(\frac{1}{q} + q\right)^2 + 3 q^5 \left(\frac{1}{q} + q\right)^2 - q^6 \left(\frac{1}{q} + q\right)^3$

Simplify[t5]

$q + q^3 + q^5 - q^9$

t1 = Xm[1, 12, 2, 13] Xm[7, 2, 8, 3] Xm[3, 8, 4, 9] Xm[11, 4, 12, 5]

Xp[13, 7, 14, 6] Xp[9, 15, 10, 14] Xp[15, 11, 16, 10] Xp[5, 1, 6, 16]

Xm[1, 12, 2, 13] Xm[3, 8, 4, 9] Xm[7, 2, 8, 3] Xm[11, 4, 12, 5]

Xp[5, 1, 6, 16] Xp[9, 15, 10, 14] Xp[13, 7, 14, 6] Xp[15, 11, 16, 10]

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t2 = t1 /. {
  Xp[i_, j_, k_, l_] := q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
  Xm[i_, j_, k_, l_] := -q^(-2) * P[i, j] P[k, l] + 1/q * P[i, l] P[j, k]
}
(-q^2 P[1, 6] P[5, 16] + q P[5, 1] P[6, 16])
( (P[2, 8] P[7, 3] / q - P[7, 2] P[8, 3] / q^2) ( -P[3, 8] P[4, 9] / q^2 + P[3, 9] P[8, 4] / q ) )
( -P[1, 12] P[2, 13] / q^2 + P[1, 13] P[12, 2] / q ) ( P[4, 12] P[11, 5] / q - P[11, 4] P[12, 5] / q^2 )
(-q^2 P[7, 14] P[13, 6] + q P[13, 7] P[14, 6]) (q P[9, 15] P[10, 14] - q^2 P[9, 14] P[15, 10])
(-q^2 P[11, 16] P[15, 10] + q P[15, 11] P[16, 10])
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t3 = Expand[t2]
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-q P[1, 6] P[1, 12] P[2, 8] P[2, 13] P[3, 8] ... 5 ...
 P[9, 15] P[10, 14] P[11, 5] P[11, 16] P[13, 6] P[15, 10] + ... 382 ...

large output
show less
show more
show all
set size limit...

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SetAttributes[P, Orderless]
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Short[t4 = t3 //. P[a_, b_] P[b_, c_] := P[a, c], 15]
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q^4 P[6, 13]^2 P[8, 12]^2 P[9, 14]^2 P[10, 15]^2 P[11, 16]^2 -
q^3 P[9, 14]^2 P[10, 15]^2 P[11, 16]^2 P[12, 13]^2 -
2 q^3 P[6, 13]^2 P[10, 15]^2 P[11, 16]^2 P[12, 14]^2 +
q^2 P[3, 8]^2 P[6, 13]^2 P[10, 15]^2 P[11, 16]^2 P[12, 14]^2 -
q P[7, 13]^2 P[10, 15]^2 P[11, 16]^2 P[12, 14]^2 + <<139>> +
P[8, 13]^2 P[12, 14]^2 P[15, 16]^2 - 6 q P[13, 14]^2 P[15, 16]^2 +
2 P[3, 8]^2 P[13, 14]^2 P[15, 16]^2 + q^2 P[8, 12]^2 P[13, 14]^2 P[15, 16]^2
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```
t5 = t4 /. {P[i_, i_] := (q+1/q), P[i_, j_]^2 := (q+1/q)}
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37 (1/q + q) - 47 (1/q + q)^2 / q - 47 q (1/q + q)^2 + 32 (1/q + q)^3 +
27 (1/q + q)^3 / q^2 + 27 q^2 (1/q + q)^3 - 8 (1/q + q)^4 / q^3 - 9 (1/q + q)^4 / q - 9 q (1/q + q)^4 -
8 q^3 (1/q + q)^4 + (1/q + q)^5 + (1/q + q)^5 / q^4 + (1/q + q)^5 / q^2 + q^2 (1/q + q)^5 + q^4 (1/q + q)^5
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Simplify[t5]
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1/q^9 - 2/q^7 + 2/q^5 - 1/q^3 + 1/q + q - q^3 + 2 q^5 - 2 q^7 + q^9
```

$$1 + 1$$

2