

# The Torus Tunnel Bridge Challenge

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(*Parameterize the two circles which determine the torus*)
x[r_,  $\theta$ _] := r Cos[ $\theta$ ] ;
y[r_,  $\theta$ _] := r Sin[ $\theta$ ] ;
w[R_,  $\rho$ _] := R Cos[ $\rho$ ] ;
z[R_,  $\rho$ _] := R Sin[ $\rho$ ] ;

(*Parametrization of the torus in 4D*)
Torus[r_, R_,  $\theta$ _,  $\rho$ _] := {x[r,  $\theta$ ], y[r,  $\theta$ ], w[R,  $\rho$ ], z[R,  $\rho$ ]}

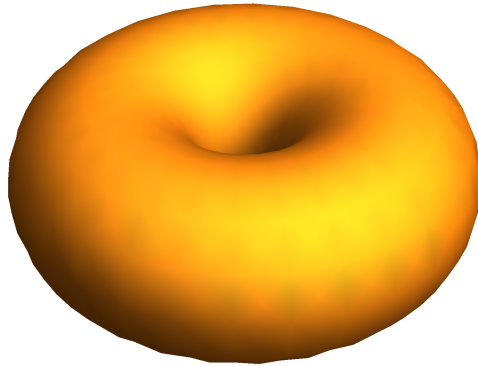
(*Project this torus into 3D*)
StereographicProjection[{x_, y_, z_, w_}] := {x, y, z} / (1 - w)

(*Useful clean plotting function*)
PlotTorus[torus_] := ParametricPlot3D[torus,
  { $\theta$ , 0, 2  $\pi$ }, { $\rho$ , 0, 2  $\pi$ }, Axes  $\rightarrow$  False, Boxed  $\rightarrow$  False, Mesh  $\rightarrow$  None]

(*Play with the radii defining the torus*)
Manipulate[PlotTorus[StereographicProjection@Torus[r, R,  $\theta$ ,  $\rho$ ]],
  {{r, 1/Sqrt[2]}, 0.1, 1}, {{R, 1/Sqrt[2]}, 0.1, 1}]
```



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(*Define the Clifford torus,
with symmetric properties which will make later pictures clearer*)
CliffordTorus[ $\theta$ _,  $\rho$ _] := Torus[1/Sqrt[2], 1/Sqrt[2],  $\theta$ ,  $\rho$ ]
PlotTorus[StereographicProjection@CliffordTorus[ $\theta$ ,  $\rho$ ]]
```



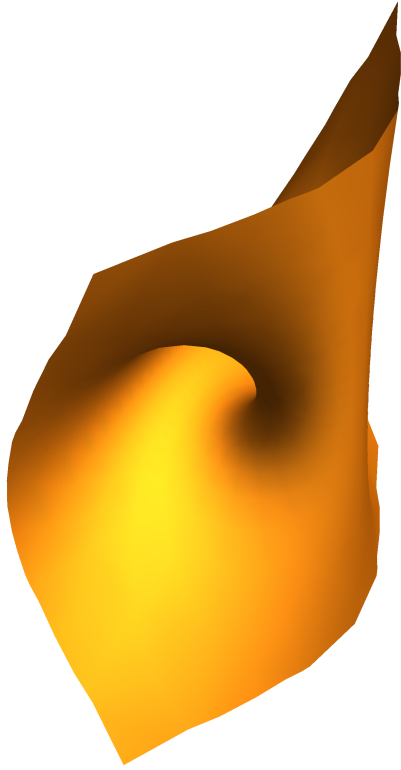
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(* Define a few rotations in 4D space *)
Rotxw[ $txw$ _] := RotationTransform[ $txw$ , {{1, 0, 0, 0}, {0, 0, 1, 0}}]
Rotwz[ $twz$ _] := RotationTransform[ $twz$ , {{0, 0, 1, 0}, {0, 0, 0, 1}}]
Rotxz[ $txz$ _] := RotationTransform[ $txz$ , {{1, 0, 0, 0}, {0, 0, 0, 1}}]

(*Play around with these rotations*)
Manipulate[PlotTorus[
  StereographicProjection[Rotxz[ $Txz$ ]@Rotxw[ $Txw$ ]@Rotwz[ $Twz$ ]@CliffordTorus[ $\theta$ ,  $\rho$ ]],
  { $Twz$ , 0, 2 Pi}, { $Txw$ , 0, 2 Pi}, { $Txz$ , 0, 2 Pi}]
```

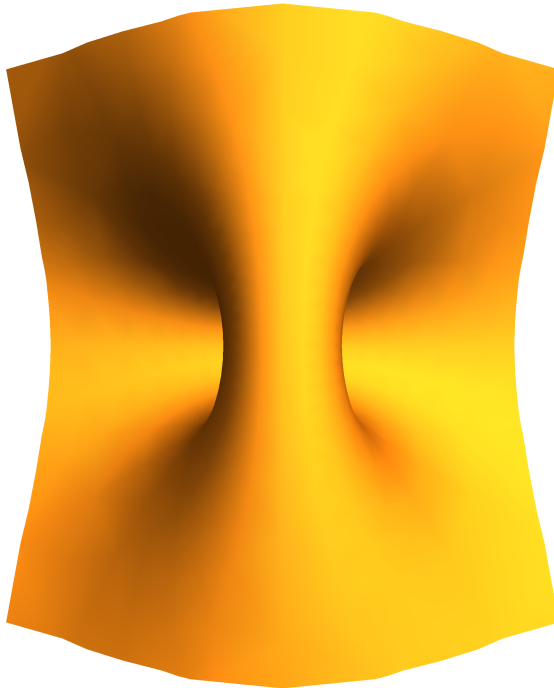


(\*If you rotate about the xz-axis by  $\frac{\pi}{4}$  we get the desired tunnel bridge image\*)

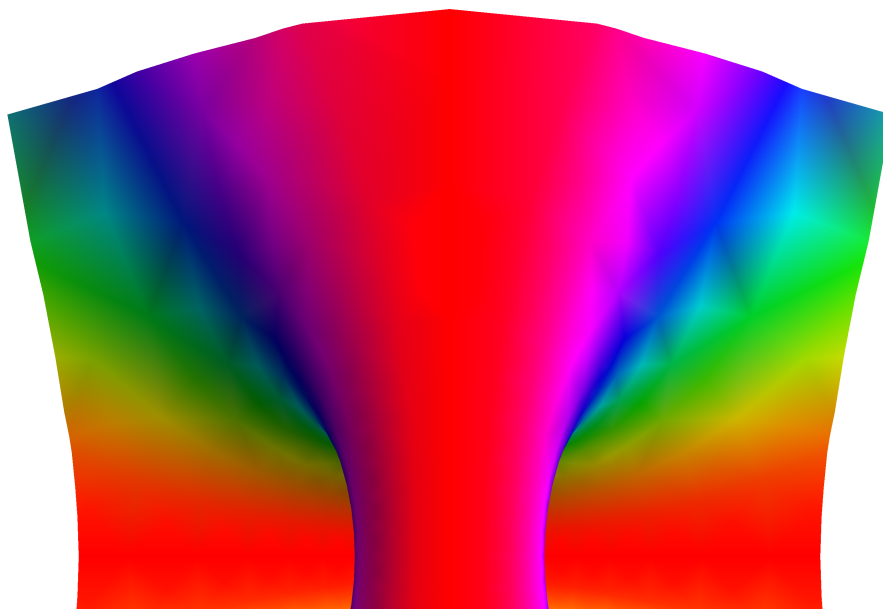
```
PlotTorus[StereographicProjection[Rotxz[Pi / 4]@N@CliffordTorus[ $\theta$ ,  $\rho$ ]]]
```



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(*We can visualize this from above the tunnel, from the point {1,0,0} *)
tunnel = StereographicProjection[Rotxz[Pi / 4]@N@CliffordTorus[ $\theta$ ,  $\rho$ ]];
ParametricPlot3D[tunnel, { $\theta$ , 0, 2  $\pi$ }, { $\rho$ , 0, 2  $\pi$ },
  Axes  $\rightarrow$  False, Boxed  $\rightarrow$  False, Mesh  $\rightarrow$  None, ViewPoint  $\rightarrow$  {1, 0, 0}]
```

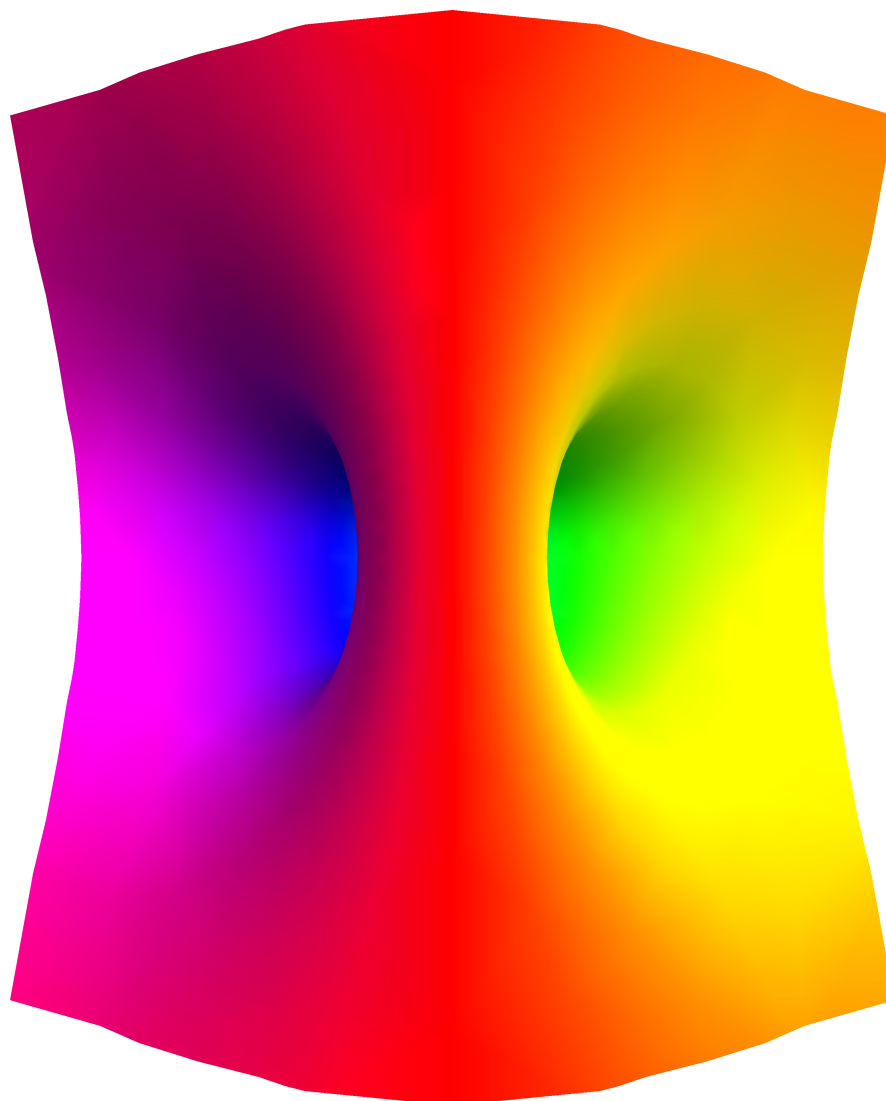
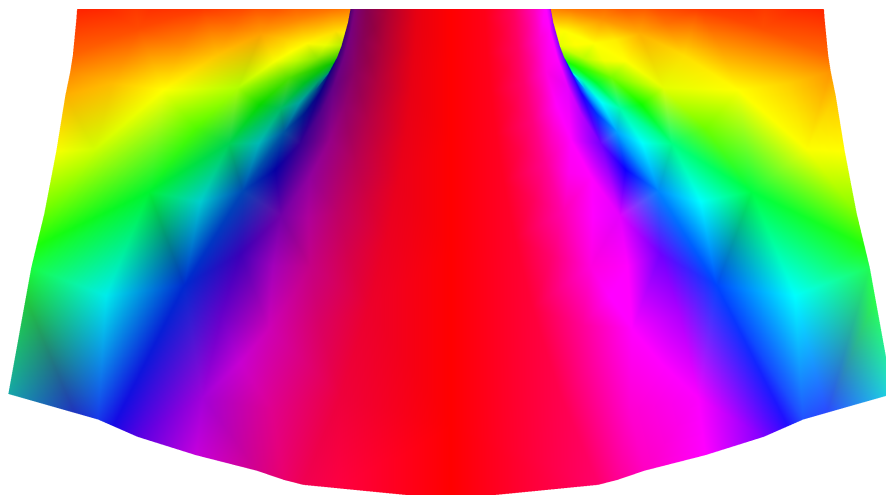


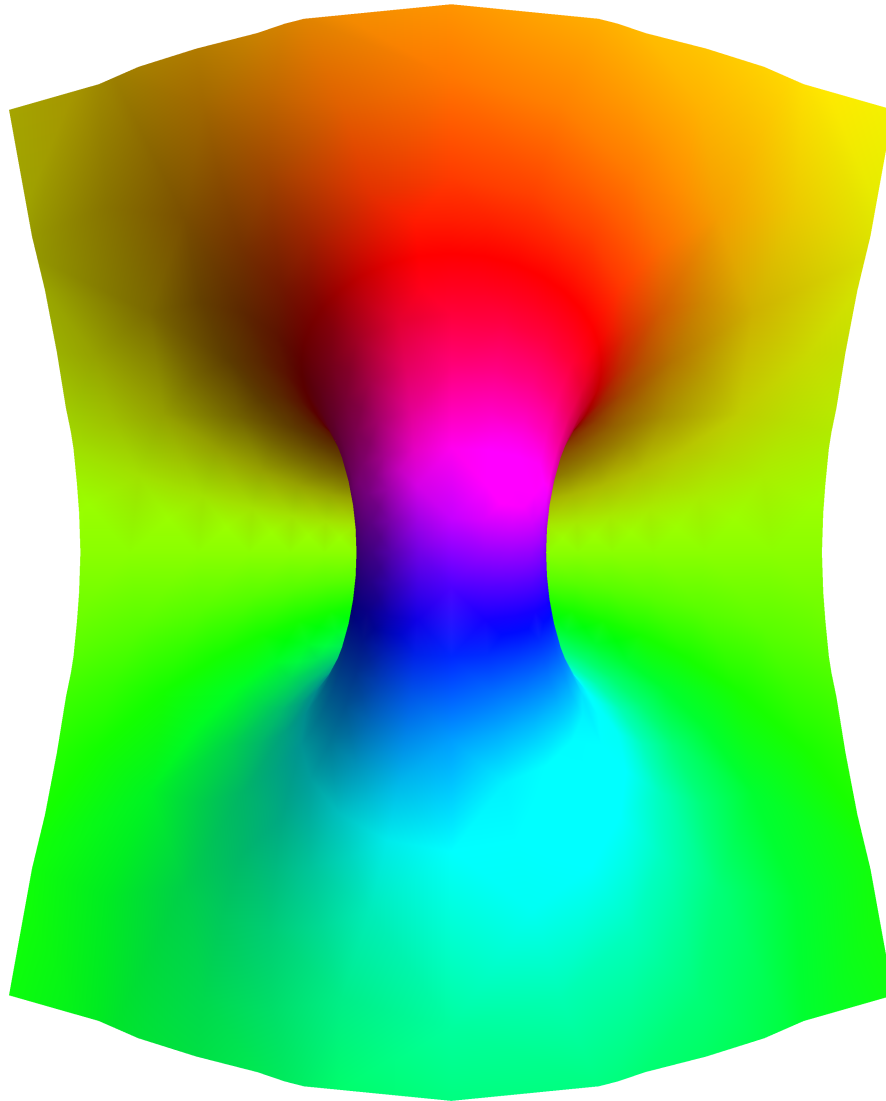
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(*Here are some prettier colourings of this*)
Table[ParametricPlot3D[tunnel, { $\theta$ , 0, 2  $\pi$ }, { $\rho$ , 0, 2  $\pi$ }, Axes  $\rightarrow$  False, Boxed  $\rightarrow$  False,
  Mesh  $\rightarrow$  None, ViewPoint  $\rightarrow$  {1, 0, 0}, ColorFunction  $\rightarrow$  Function[{{x, y, z,  $\theta$ ,  $\rho$ }, i],
  ImageSize  $\rightarrow$  Large], {i, {Hue[x], Hue[ $\theta$ ], Hue[ $\rho$ ]}}]
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