# Thursday Jan 22, hours 8-9: Formulate an Equivalent Problem

January-22-15 9:03 AM

**1.3.1.** Find a general formula for the $n$th derivation of $f(x) = 1/(1-x^2)$.

Trick: always use partial fractions.

Another example: Compute $\int \frac{dx}{1-x^2}$

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**1.3.2.** Find all solutions of $x^4 + x^3 + x^2 + x + 1 = 0$.

Method 1. use roots of unity.

Method 2. If $\lambda$ solves, so does $\frac{1}{\lambda}$. Make it manifest.

$$f = x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2} = 0$$

Write $f$ in terms of $x + \frac{1}{x}$, $x \cdot \frac{1}{x}$, ...

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**1.3.3.** $P$ is a point inside a given triangle $ABC$; $D, E, F$ are the feet of the perpendiculars from $P$ to the lines $BC$, $CA$, $AB$, respectively. Find all $P$ for which

$$\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$$

is minimal.

Solve as in book, then ask students for a rationalization.

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**1.3.5.** On a circle $n$ points are selected and the chords joining them in pairs are drawn. Assuming that no three of these chords are concurrent (except at the endpoints), how many points of intersection are there?

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**1.3.6.** Given a positive integer $n$, find the number of quadruples of integers $(a, b, c, d)$ such that $0 \leqslant a \leqslant b \leqslant c \leqslant d \leqslant n$.

Is there a nearby problem we know how to solve?

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**1.3.7.** The number 5 can be expressed as a sum of 3 natural numbers, taking order into account, in 6 ways, namely, as $5 = 1 + 1 + 3 = 1 + 3 + 1 = 3 + 1 + 1 = 1 + 2 + 2 = 2 + 1 + 2 = 2 + 2 + 1$. Let $m$ and $n$ be natural numbers such that $m \leqslant n$. In how many ways can $n$ be written as a sum of $m$ natural numbers, taking order into account?

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Then same Q with "natural" replaced with "non-negative integer".