Pensieve header: A constrained minimation for the hard part of the isoperimetric inequallity.

$$f = \frac{1}{2} a\, b\, \mathrm{Sin}[\alpha];$$

$$c = a + b - \sqrt{a^2 + b^2 - 2\, a\, b\, \mathrm{Cos}[\alpha]}\;;$$

$$g = f - \lambda\, c;$$

$$\mathrm{grad} = \{\partial_a g,\ \partial_b g\}$$

$$\left\{-\lambda \left(1 - \frac{2\,a - 2\,b\,\mathrm{Cos}[\alpha]}{2\sqrt{a^2 + b^2 - 2\,a\,b\,\mathrm{Cos}[\alpha]}}\right) + \frac{1}{2} b\,\mathrm{Sin}[\alpha],\ -\lambda \left(1 - \frac{2\,b - 2\,a\,\mathrm{Cos}[\alpha]}{2\sqrt{a^2 + b^2 - 2\,a\,b\,\mathrm{Cos}[\alpha]}}\right) + \frac{1}{2} a\,\mathrm{Sin}[\alpha]\right\}$$

$$\mathrm{sols} = \mathrm{Solve}[\mathrm{grad} == \{0, 0\},\ \{a, b\}]$$

$$\left\{\left\{a \to 2\,\lambda\,\mathrm{Cot}\left[\frac{\alpha}{2}\right],\ b \to 0\right\},\ \left\{a \to 0,\ b \to 2\,\lambda\,\mathrm{Cot}\left[\frac{\alpha}{2}\right]\right\},\ \left\{a \to \frac{\lambda\,\mathrm{Csc}\left[\frac{\alpha}{2}\right]\left(\mathrm{Cos}\left[\frac{\alpha}{4}\right] - \mathrm{Sin}\left[\frac{\alpha}{4}\right]\right)}{\mathrm{Cos}\left[\frac{\alpha}{4}\right] + \mathrm{Sin}\left[\frac{\alpha}{4}\right]},\ b \to \frac{\lambda\,\mathrm{Csc}\left[\frac{\alpha}{2}\right]\left(\mathrm{Cos}\left[\frac{\alpha}{4}\right] - \mathrm{Sin}\left[\frac{\alpha}{4}\right]\right)}{\mathrm{Cos}\left[\frac{\alpha}{4}\right] + \mathrm{Sin}\left[\frac{\alpha}{4}\right]}\right\},\ \left\{a \to \frac{\lambda\,\mathrm{Csc}\left[\frac{\alpha}{2}\right]\left(\mathrm{Cos}\left[\frac{\alpha}{4}\right] + \mathrm{Sin}\left[\frac{\alpha}{4}\right]\right)}{\mathrm{Cos}\left[\frac{\alpha}{4}\right] - \mathrm{Sin}\left[\frac{\alpha}{4}\right]},\ b \to \frac{\lambda\,\mathrm{Csc}\left[\frac{\alpha}{2}\right]\left(\mathrm{Cos}\left[\frac{\alpha}{4}\right] + \mathrm{Sin}\left[\frac{\alpha}{4}\right]\right)}{\mathrm{Cos}\left[\frac{\alpha}{4}\right] - \mathrm{Sin}\left[\frac{\alpha}{4}\right]}\right\}\right\}$$

$$a == b\ /.\ \mathrm{sols}$$

$$\left\{2\,\lambda\,\mathrm{Cot}\left[\frac{\alpha}{2}\right] == 0,\ 0 == 2\,\lambda\,\mathrm{Cot}\left[\frac{\alpha}{2}\right],\ \mathrm{True},\ \mathrm{True}\right\}$$