

Name (Last, First): \_\_\_\_\_

Student ID: \_\_\_\_\_

Dror Bar-Natan: Classes: 2014-15: MAT 475 Problem Solving Seminar:

<http://drorbn.net/15-475>

**Quiz 9** on March 17, 2015: “Pursue Parity”. You have 25 minutes to solve two of the three problems below. Please write on both sides of the page. **Good Luck!**

**Marking Comment.** My decision remains to simplify the management of this course and mark the quizzes myself, though at a delay of one week, in a symbolic acknowledgement of the ongoing TA strike.

**Problem 1.** Can you cover an  $8 \times 8$  chessboard with 21 rectangles of size  $3 \times 1$  and a single extra  $1 \times 1$  square? If you can, what chessboard positions might the  $1 \times 1$  square occupy? (You need to justify your assertions, of course). For reference, in the chessboard on the right, the rook is at position g4.

**Problem 2** (Larson’s 1.10.1, reworded). Given 9 distinct points in  $\mathbb{Z}^3$ , show that there is some point in  $\mathbb{Z}^3$  which is exactly half way between two of these 9 points.

**Problem 3** (Larson’s 1.10.10, reworded). Show that for every positive integer  $a$ , the equation  $x^2 - y^2 = a^3$  has solutions with  $x, y \in \mathbb{Z}$ .

