Name (Last, First): ______________________________ Student ID: ____________________

Quiz 8 on March 10, 2015: "argue by contradiction" and the Pigeonhole Principle. You have 30 minutes to solve the two problems below. Please write on both sides of the page. **Good Luck!**

Marking Comment. At least for now, I have decided to simplify the management of this course and mark the quizzes myself, though at a delay of one week, in symbolic acknowledgement of the ongoing TA strike.

Problem 1 (Larson's 2.6.10, expanded).

1. (4 points) Let X be any real number. Prove that among the numbers

$$X,\ 2X,\ \ldots,\ (n-1)X$$

there is at least one that differs from an integer by at most $1/n$.

2. (4 points) Let α be an irrational number. Prove that the set $\{\{n\alpha\}\colon n \in \mathbb{Z}\}$ is dense in the unit interval $[0, 1]$, where for a real number x, $\{x\}$ denotes its "fractional part" — the difference between x and the largest integer $\lfloor x \rfloor$ smaller or equal to x.

Problem 2 (5 points, Larson's 1.9.5, abbreviated). A set S of rational numbers is closed under addition and multiplication, and it is given that for every $r \in \mathbb{Q}$ exactly one of the following is true: $r \in S$, $-r \in S$, $r = 0$. Prove that S is the set of all positive rational numbers.