Name (Last, First): ________________________________ Student ID: ______________________

Dror Bar-Natan: Classes: 2014-15: MAT 475 Problem Solving Seminar: http://drorbn.net/15-475

Quiz 7 on March 3, 2015: "Divide into Cases", "Work Backwards", the Pigeonhole Principle. You have 30 minutes to solve the two problems below. Please write on both sides of the page. **Good Luck!**

Problem 1 (Larson's 2.5.13).

- (4 points) A derangement is a permutation $\sigma \in S_n$ such that for every i, $\sigma i \neq i$. Let g_n be the number of derangements in S_n. Show that

  $$g_1 = 0, \qquad g_2 = 1, \qquad g_n = (n-1)(g_{n-1} + g_{n-2}).$$

  Hint. A derangement interchanges 1 with some other element, or not.

- (4 points) Let f_n be the number of permutations in S_n that have exactly one fixed point (namely, exactly one i such that $\sigma i = i$). Show that $|f_n - g_n| = 1$.

Problem 2 (Larson's 2.6.1, 5 points). Given $n + 1$ positive integers, none of which exceeds $2n$, show that one of these integers divides another of these integers.