Name (Last, First): ______________________________ Student ID: ____________________

Dror Bar-Natan: Classes: 2014-15: MAT 475 Problem Solving Seminar: http://drorbn.net/15-475

Quiz 3 "Formulate an Equivalent Problem", January 27, 2015. You have 25 minutes to solve as much as you can of the following problems. Please write on both sides of the page. **Good Luck!**

Problem 1. In how many ways can a natural number n be written as a sum of k non-negative integers, taking order into account? For example, if $n = 2$ and $k = 3$, there are 6 ways: $2 = 2+0+0 = 0+2+0 = 0+0+2 = 0+1+1 = 1+0+1 = 1+1+0$.

Problem 2 (Larson's 1.3.15). Use a counting argument to prove that for integers $0 < r \leq n$,

$$\binom{r}{r}+\binom{r+1}{r}+\binom{r+2}{r}+\cdots+\binom{n}{r}=\binom{n+1}{r+1}.$$

Problem 3 (no credit, yet the best solutions will be advertised). What is your favourite "Formulate an Equivalent Problem" problem?

Problem 3 solution by Alessandra:

3. Show that $1+x+x^2+x^5+x^6+x^7$ has no roots greater than 1.

↳ substitute $y+1=x$ into x, and then it appears that all coefficients are positive, so the equation will never equal 0, and thus the original equation will also never equal zero.

∴ no roots greater than 1.

Problem 3 solution by Yizhou:

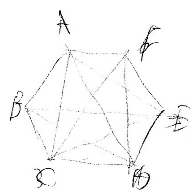
Problem 3 shake hand problem

for 6 people,

there are at least 3 among whom each two people have shaken hand or no one has shaken hand with another

$\Updownarrow$

2-coloring of the full graph



each edge is colored Red or Blue

the result is equivalent to "there must be a red triangle or a blue triangle"

for A, AB, AC, AD, AE, AF

at least $\left[\frac{5}{2}\right]+1=3$ of them are same color

WLOG AB, AC, AD red

if at least one of BC BD or CD red, red △. done

otherwise, BC, BD, CD all blue $\Rightarrow$ △BCD blue done.