15-344 Combinatorics on Sep 24 - hour 6: Planarity

Thursday, September 17, 2015 7:59 PM

HW1 is on web.
Read Along: Sect 1.4.

Def Path in G: $(x_0, \dots, x_n)$ s.t. $\forall\ 1 \le i \le n$ $(x_{i-1}, x_i) \in E$.

Length $:= n$ = "number of hops".
Cycle: path s.t. $x_0 = x_n$.

Thm The complete graph K_5 and the complete bipartite graph $K_{3,3}$ are not

on board.

The utilities problem.
Proof that $K_{3,3}$ is not planar using "circle chord".

If G is placed in the plane, "face"/"region"

Euler's Thm $v - e + f = 2$ for a connected plane graph.

By induction on $v+e$: not needed? but "no isolated" is needed.
If G has a bivalent or a univalent vertex, easy. Otherwise it has a cycle; remove one edge from it. $\square$

done line

better to introduce $\chi(G) = v - e + f$ so it can be used throughout the proof.

must say "after removal"

Corollary If G is a connected planar graph, $e \le 3v - 6$

Proof $3f \le 2e$ so $3f + p = 2e$ $p \ge 0$

$$v - e + f = 2 \Rightarrow v - e + \frac{2e - p}{3} = 2$$

so $v - \frac{1}{3}e = 2 + \frac{p}{3}$

so $3v - e = 6 + p$ or $3v - e \ge 6$ $\square$

Now if K_5 was planar

$$3 \cdot 5 - 10 \ge 6 \quad \Rightarrow\Leftarrow.$$

Kuratowski's thm

graph remains connected; also in the case of a cycle.