15-344 Combinatorics on Sep 22 - hours 4-5: Edge Counting

Thursday, September 17, 2015 7:59 PM

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Read Along: Sects. 1.3-1.4. } on board.

Thm In any G, the sum of the degrees of all vertices is twice the number of edges.

Examples: 1. Def K_n; how many edges?

2. Is there a group of 7 people, each of which is FB friend w/ exactly 3 in the group?

⇒ In any G, number of odd-valency verts is even.

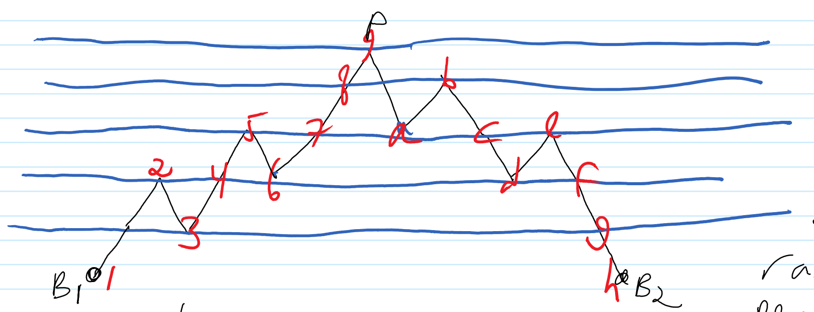
Paths and cycles in graphs

Def A path in a graph.

A connected graph, A connected component

Every graph is a union of its connected comps.

A mountain-climbing problem:



Can two climbers, starting from the two sides of a mountain range, reach the peak at the same time while always being at the same height?

Assumptions:

1. One peak, higher than anything else

2. Two base camps on both sides, lower than anything else.

Sol'n: construct a graph $G=(V,E)$ as follows:

V: ordered pairs (L,R) where L & R are on the mountain, $L\in[B,P]$ $R\in[P,B_2]$ and at least one is a local max ($\wedge$) or a local min ($\vee$) or a base or the peak.

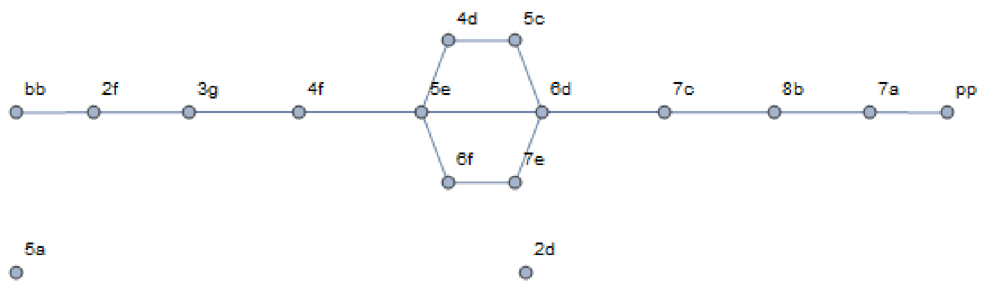
(bb) (1h) (2d) (2f) (3g) (4d) (4f) (5a) (5c) (5e) (6d) (6f) (7a) (7c) 7e, 8b, pp

E: Direct paths.

from MountainClimbing.nb

```
In[2]:= Graph[
  {"bb", "2d", "2f", "3g", "4d", "4f", "5a", "5c", "5e", "6d", "6f", "7a",
   "7c", "7e", "8b", "pp"},
  {"bb" ↔ "2f", "2f" ↔ "3g", "3g" ↔ "4f", "4d" ↔ "5c", "4d" ↔ "5e",
   "4f" ↔ "5e", "5c" ↔ "6d", "5e" ↔ "6d", "5e" ↔ "6f", "6d" ↔ "7c",
   "6d" ↔ "7e", "6f" ↔ "7e", "7a" ↔ "pp", "7a" ↔ "8b", "7c" ↔ "8b"},
  VertexLabels → "Name"
 ]
```

Out[2]=



Def a cycle in a graph

Def Bipartite graph

Thm G is bipartite iff every cycle in G is of even length.

Thm The complete graph K_5 and the complete bipartite graph $K_{3,3}$ are not planar.

done line.

planar.

done line.

<u>Def</u> complete bipartite $K_{m,n}$ $(|E| = m \cdot n)$

<u>Def</u> "Planar"

The utilities problem.

proof that $K_{3,3}$ is not planar.

If G is placed in the plane, "face"/"region"

Euler's Thm $v - e + f = 2$ for a connected plane graph.

<u>Corollary</u> If G is a connected planar graph, $e \leq 3v - 6$

<u>Proof</u> $3f \leq 2e$ so $3f + p = 2e$ $p \geq 0$

$$v - e + f = 2 \Rightarrow v - e + \frac{2e - p}{3} = 2$$

so $v - \frac{1}{3}e = 2 + \frac{p}{3}$

so $3v - e = 6 + p$ or $3v - e \geq 6$ $\square$

Now if K_5 was planar

$$3 \cdot 5 - 10 \geq 6 \Rightarrow \Leftarrow.$$

Kuratowski's Thm