Dror Bar-Natan: Academic Pensieve: Classes: 15-344-Combinatorics:

15-344 Combinatorics on Sep 17 - hour 3: Isomorphism, Edge Counting

Tuesday, September 15, 2015   5:46 PM

All who asked for a wiki account should have received it!

Riddle Along. 1 2 3 4 5 6 7 8 9

Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second?

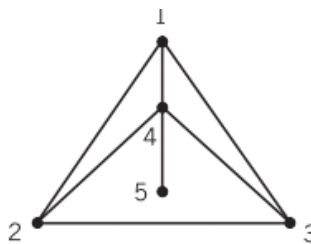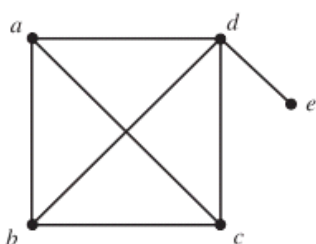Read Along. Textbook sections 1.2 & 1.3.
Consider MAT 332!
Tutorials Today! 4-5 & 5-6 @ LM 158
Handout here & on web!
Today I'll vanish @ 3 sharp; sorry,

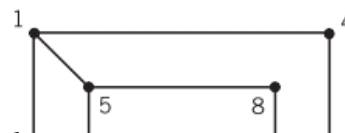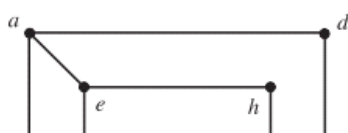Def "G_1 is isomorphic to G_2"

Example.

$$e \to 5$$
$$d \to 4$$
$$a \to 2$$
$$c \to 3$$
$$b \to 1$$

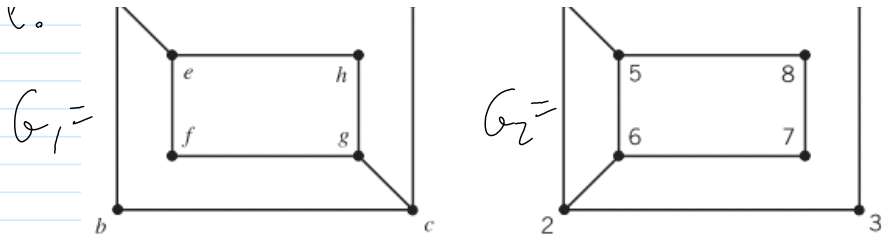[clearly this is a reflexive, symmetric
 & transitive relation]

If two graphs are isomorphic, they have the same
1. $|V|$
2. $|E|$
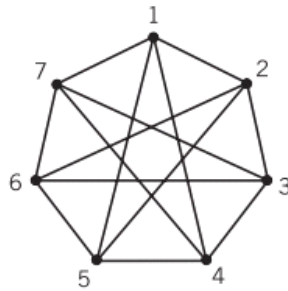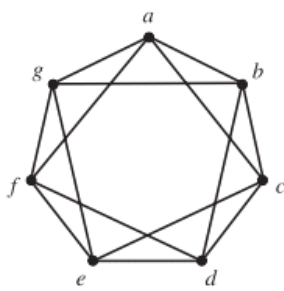3. Vertex degrees/Valencies
4. Subgraphs

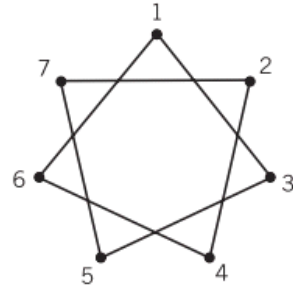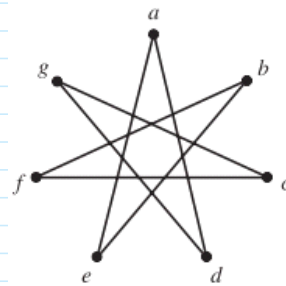Example.

Example.

$G_1 =$

$G_2 =$

Not isomorphic! G_2 has a pair of
bivalent vertices connected by an edge
while G_1 doesn't.

There is no efficient algorithm for graph
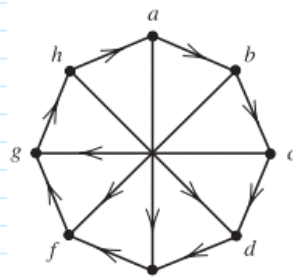isomorphism! So often cleverness is
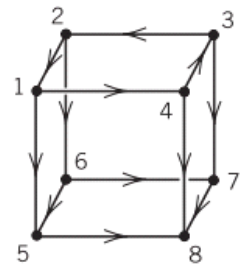involved. Example:

$a \longrightarrow 1$

The complement graph:

Example: Directed graphs:

has directed
cycle

doesn't.

done
fine

__Thm__ In any G, the sum of the degrees
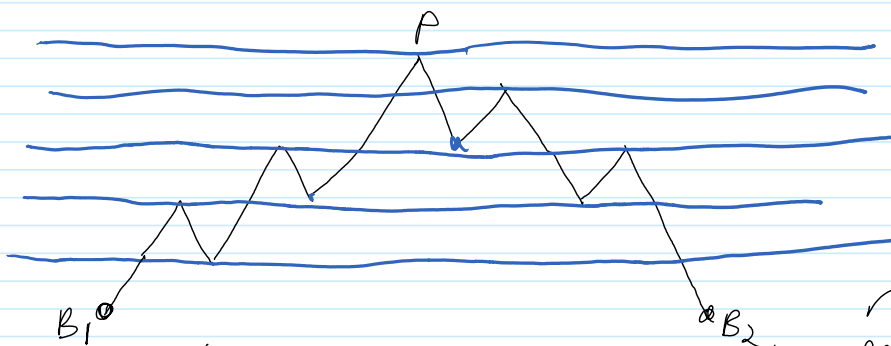of all vertices is twice the number

of edges.

Examples: 1. Def K_n; how many edges?

2. Is there a group of 7 people, each of which is FB friend w/ exactly 3 in the group?

⟹ In any G, number of odd-valency verts is even.

Human—wolf—goat—river problems. } mentioned

A mountain-climbing problem:

Can two climbers, starting from the two sides of a mountain range, reach the peak at the same time while always being at the same height?

Assumptions:

1. One peak, higher than anything else

2. Two base camps on both sides, lower than anything else.