15-344 Combinatorics on Oct 6 - hours 10-11: More Hamilton circuits, graph colourings

Thursday, September 17, 2015 7:59 PM

Class photo on web. Add your name!

Read Along: Sects 2.2 – 2.4.

Def A Gray code is a function $g:\{0,\dots,2^n-1\}\to\{0,1\}^n$ s.t. $\forall i$ $f(i)$ & $f(i+1)$ differ by at most one digit. on board

Defn: $Q_n=(V_n,E_n)$, where $V_n=\{0,1\}^n$ and

$E_n=\{(ll/*\to 0, ll/*\to 1) : ll\in\{0,1,*\}^n \text{ has one } *\}$ (show graphs)

Thm Gray code $\Leftrightarrow$ Hamilton path in the n-dim cube graph Q_n.

Thm Q_n has a Hamilton circuit.

Proof By induction. Let γ be a Hamilton circuit in Q_{n-1}, let $\tilde{e}=(V_0V_1)$ be the first edge in it, and let γ' be γ minus the first edge. Then $(\gamma'0, \overleftarrow{\gamma'}1, V_1 0)$ is a Hamilton circuit in Q_n (show circuits)

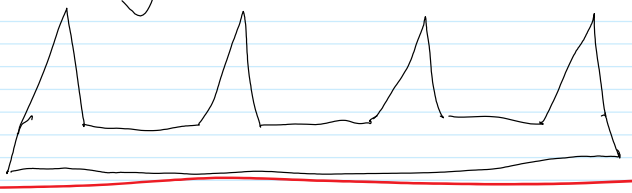
The four-colour problem.
Graph vertex-colouring
The dual graph construction.

The chromatic number.
Example: The chromatic # of: K_n, $K_{n,m}$, Q_n

Thm The vertices in a triangulation of

Thm The vertices in a triangulation of a polygon can be 3-colored.

The art-gallery problem.

Example 

stated, not done.

undone below

Thm (the 5-colour thm) every planar graph can be 5-coloured.

Lemma A planar graph has at least one vertex of valency ≤ 5.

pf of lemma $e \leq 3V-6$ so $2e \leq 6V-12$

so $\sum \deg(v) \leq 6V-12$ so $\sum(\deg(v)-6) \leq -12$

so at least one of these numbers is negative.

pf of thm. (by induction.....)

Warning. In the rest of this class I may be telling lies.

pf of the 4CT.

Cases: