15-344 Combinatorics on Oct 22, hour 18: Dijkstra's Algorithm

Thursday, September 17, 2015 7:59 PM

Reminder: Term exam in 2 locations!

Read Along: Sects 4.1, 4.2.

Network: Connected G with $w: E \to \mathbb{R}_{\geq 0}$.

1. The Fastest way from a to z?
2. The Fastest H-circuit?
3. The cheapest spanning tree?
4. Capacity from a to z?

on board.

Sol'n of "Fastest from a to z":
variant of "Dijkstra's algorithm"

Throughout, we'll have a decomposition

$V = C \sqcup U$, a function $m: V \to \mathbb{R}_+ \cup \{\infty\}$,

and a partially defined function $b: V \to V$;

C: "confirmed" U: "unknown"

C will grow, U will shrink, until $z \in C$.

start w/ $C = \{a\}$, $U = V - \{a\}$

$m(a) = 0$ $b(a) = \text{undef}$; for all x adjacent to a, set

$m(x) = w(ax)$ $b(x) = a$

At each further step, let x_0 be where m atains its minimum on U, move x_0 from U to C, and for each neighbor $y \in U$ of x_0,

if $m(x_0) + w(x_0 y) \geq m(y)$, do nothing

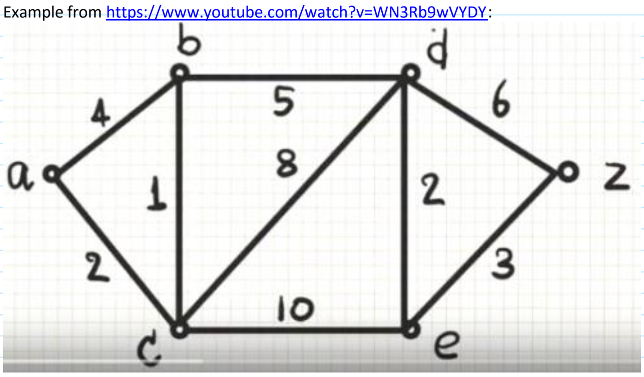
else set $m(y) \leftarrow m(x_0) + w(x_0 y)$ and $b(y) = x_0$.

Claim when $z \in C$, $m(z)$ is the length of a minimal path to z, and a minimal path can be found by backtracking from z using b.

Proof Inductive assertion: After each step, assertion is true for any $z \in C$, and also for any $z \in V$ s.t. $m(z) < \infty$, provided we restrict to paths that are in C in the one-before-the-last step.

not done

Example from https://www.youtube.com/watch?v=WN3Rb9wVYDY:



Efficiency estimate: for a graph with V vertices, each of valency ≤ 10, about $V(10+V)$. Can be shrunk to $V(10+\log V)$.

very sketchy

done line.

Cheapest spanning tree:

Kruskal's Algorithm start w/ $T=\emptyset$; repeatedly add to T the cheapest edge that does not form a circuit w/ edge already in T.

Prim's Algorithm start w/ $T =$ a min. edge. repeatedly add to T the cheapest edge connecting T & T^c.