15-344 Combinatorics on Oct 20, hours 16-17: More chromatic polynomials, trees

Thursday, September 17, 2015 7:59 PM

Read Along: Sects 2.4, 3.1, 4.1, 4.2.

Warning: Term exam in 2 locations!

Thm

up 1. If $(x,y) \notin E$, $P_k(G) = P_k(G_{+(x,y)}) + P_k(G_{x=y})$.

down 2. If $(x,y) \in E$, $P_k(G) = P_k(G_{-(x,y)}) - P_k(G_{x=y})$.

on board

Compute $P_k(C_5)$.

Prove Thm.

Little on trees:

Thm For a connected graph T, TFAE:

1. T has no circuits.
2. Let a be some vertex of T. For any other vertex x of T there is a unique simple path from a to x.
3. There is a unique simple path between any two $x \neq y \in V$
4. T is "minimally connected": it becomes disconnected upon the removal of any edge in T.

Pf T has circuit $\Leftrightarrow$ not unique path $\Leftrightarrow$ not min. connected.

Def Such T is a tree.

Thm In a tree, $e = v - 1$.

Def Network: A (connected) graph G together w/ a "weight function" $w: E \to \mathbb{R}_{\geq 0}$
{sometimes directed graphs, sometimes multigraphs}

Problems: 1. "The fastest way from a to z"

2. Traveling salesperson: The fastest H-circuit

3. "The cheapest spanning tree".

4. "Capacity from a to z"

done line

Sol'n of "fastest from a to z".
variant of "Dijkstra's algorithm"

Throughout, we'll have a decomposition

$V = C \sqcup U$, a function $m: V \to \mathbb{R}_+ \cup \{\infty\}$,

and a partially defn'd function $b: V \to V$;

C: "confirmed" U: "unknown"

C will grow, U will shrink, until $z \in C$.

start w/ $C = \{a\}$, $U = V - \{a\}$

$m(a) = 0$ $b(a) = $ undef; for all x adjacent to a, set

$m(x) = w(ax)$ $\quad b(x) = a$

At each further step, let x_0 be where m atains its minimum on U, move x_0 from U to C, and for each neighbor $y \in U$ of x_0,

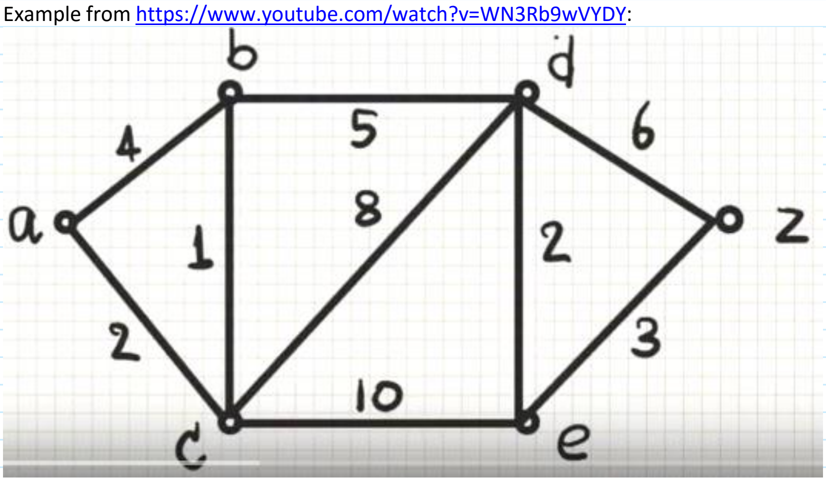
if $m(x_0) + w(x_0 y) \geq m(y)$, do nothing

else set $m(y) \leftarrow m(x_0) + w(x_0 y)$ and $b(y) = x_0$.

Claim when $z \in C$, $m(z)$ is the length of a minimal path to z, and a minimal path can be found by backtracking from z using b.

proof Inductive assertion: After each step, assertion is true for any $z \in C$, and also for any $z \in U$ s.t. $m(z) < \infty$, provided we restrict to paths that are in C in the one-before-the-last step.

Example from https://www.youtube.com/watch?v=WN3Rb9wVYDY:



Efficiency estimate: for a graph with

V vertices, each of valency ≤ 10,

about $V(10+V)$. Can be shrunk to

$V(10+\log V)$.