15-344 Combinatorics on Oct 1 - hour 9: Hamilton Circuits

Thursday, September 17, 2015 7:59 PM

class photo today!

Additional OH: Tuesdays $10^{30}-11^{30}$.

HW2 on web!

Read Along: Sects 2.1, 2.2.

Thm A multigraph G has an Euler circuit / trail (walk through all edges exactly once) iff it has no / exactly 2 vertices of odd degree.

on board

Def Hamilton circuit - a cycle that visits every vertex exactly once. Hamilton path: ...

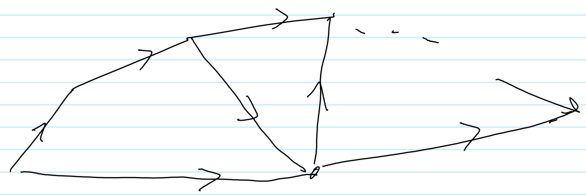
Example: City tour of Europe.

Provably no excellent way to solve!

Def A "tournament" is a directed graph obtained from K_n by choosing a direction for each edge.

Thm Every tournament has a Hamilton path.

Proof



Thm (Dirac, 1952) If a graph G has $n>2$ vertices, each of degree $\geq n/2$, then G has a Hamilton circuit.

Proof A guided exercise in tutorials.

Problem You are facing n on/off switches, presently all "off", and you know that in exactly one "code" configuration, the door opens and you can escape. What's the least number of switches necessary to cover all possibilities?

Def A Gray code is a function $g:\{0,\dots,2^n-1\}\to\{0,1\}^n$ s.t. $\forall i$ $f(i)$ & $f(i+1)$ differ by at most one digit.

Thm Gray code $\Leftrightarrow$ sol'n to problem $\Leftrightarrow$ Hamilton path in the n-dim cube graph C_n.

done line.

Thm C_n has a Hamilton path.